Session 268: Posters in Applied Mathematics

Quantifying the non-stationarity of a nonlinear bubble column

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Sandeep Rajput

Dr. Duane D. Bruns

Department of Chemical Engineering,
The University of Tennessee, Knoxville

Abstract

Two methods based on nonlinear features (return maps)are developed and implemented to compare nonlinear and chaotic system states.

In the first method, we compare the structure of data points on the return maps by comparing the means and autocovariance matrices, that capture, respectively, the shifts in mean and changes in structure.

In the second method, we utilize the power of principal curves or nonlinear PCA (NLPCA) to approximate the ‘curve’ on the return map. PDFs of Nonlinear Principal Scores (NLPS) can be compared using the classic c2 statistic. Furthermore, the NLPS reveal important information about the system dynamics (period-2 or –4, e.g.).

Comparisons of NLPS prove to be very accurate in detecting structural changes and movements. The Q statistic can also be used as a measure of dissimilarity between two process states. The methods presented are very general and can be applied to almost any time series measurements.

The methods are used for testing stationarity, and extended to cover process monitoring, event detection and fault diagnosis; and have important implications for control. In this paper we use the time series data from a nonlinear bubble column for illustration.

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Introduction

What is a bubble column?

- A liquid-filled column with gas bubbled through bottom nozzle
- Exhibits complex dynamics
  - Period-doubling/bifurcation
  - Chaotic dynamics

Why chaos?

Chaos = Higher rates of mass or heat transfer and reaction

Practical Issues

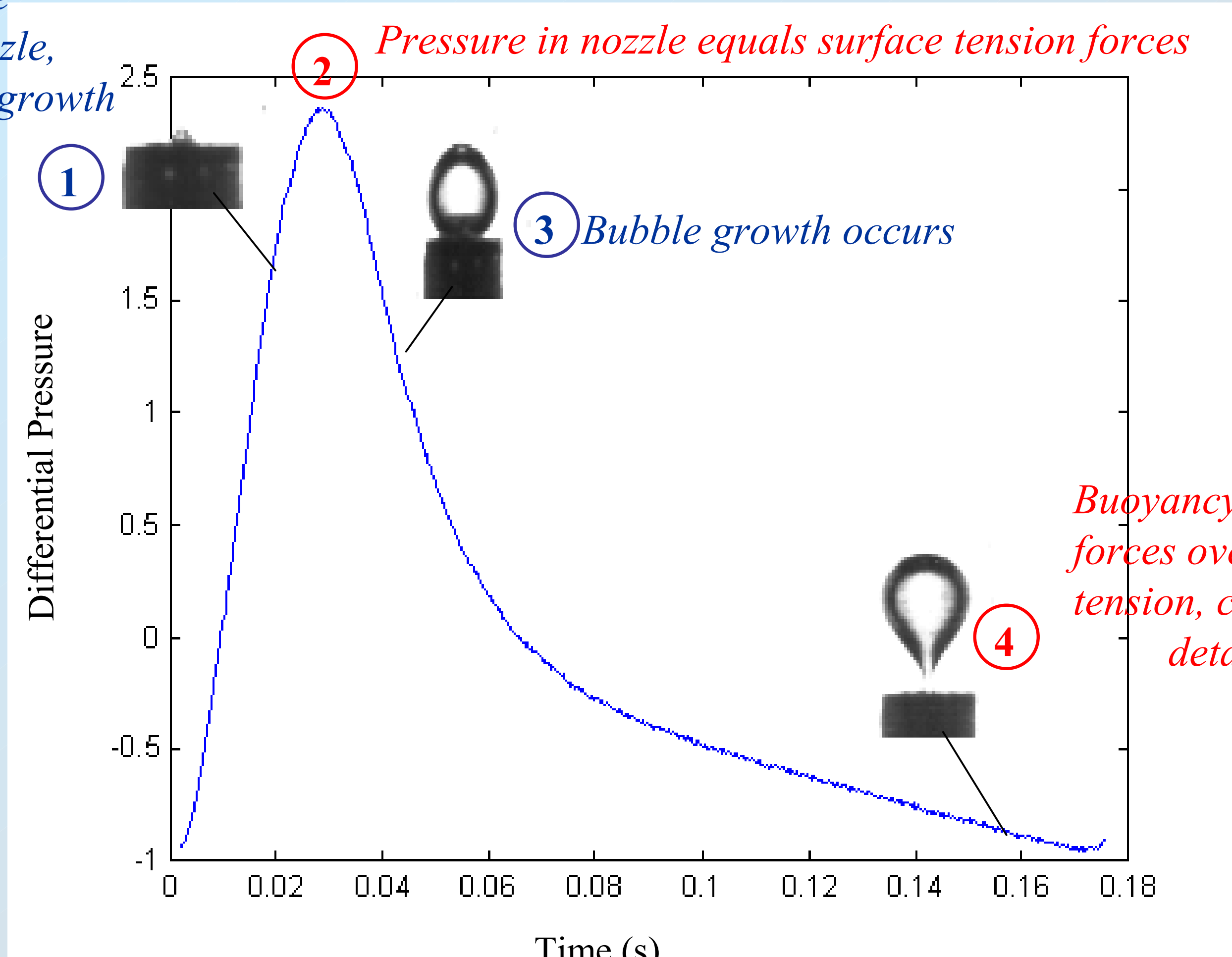
- Non-stationary and non-linear behavior
  - Dynamics changes over time (varying probabilistic measures)

Goals

- Control the system in chaotic/regular régimes
- Detect change in dynamics

The dynamics of bubbling

Surface tension forces are larger than the pressure in the nozzle, preventing bubble growth



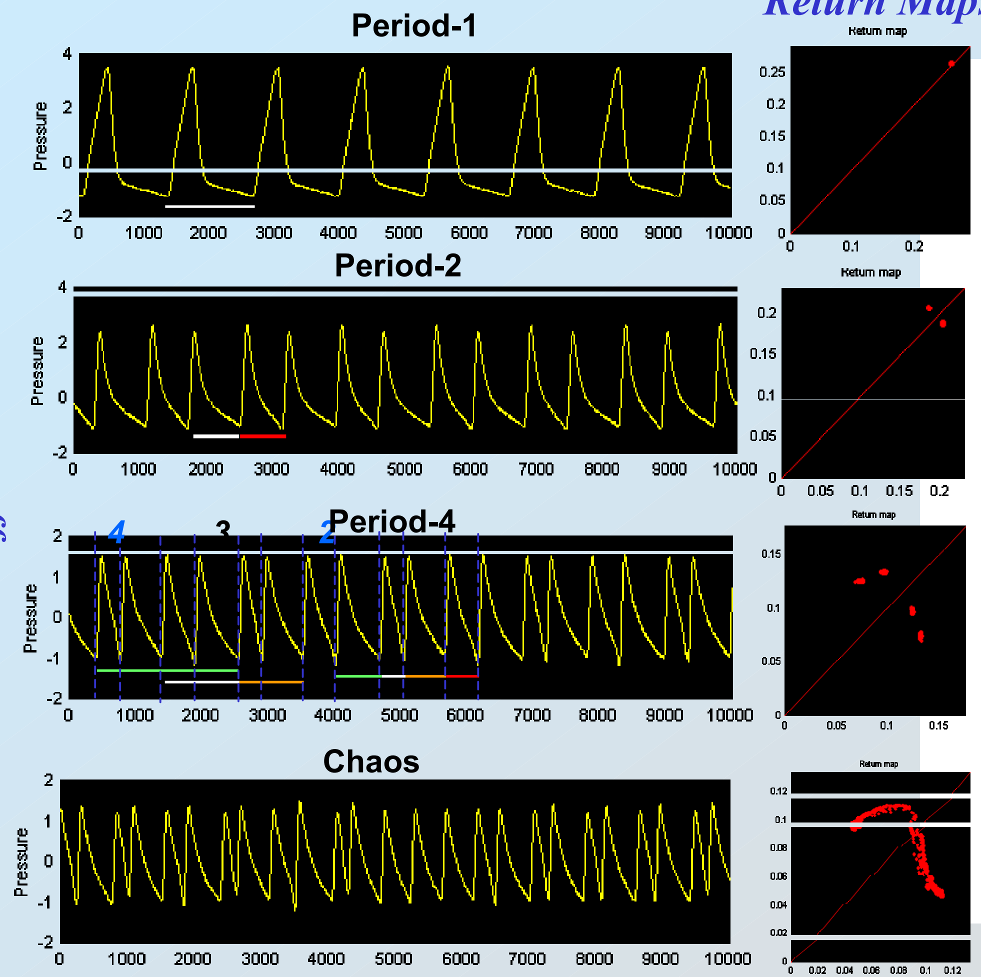
Buoyancy and inertial forces overcome surface tension, causing bubble detachment.

Photograph of high-speed images of slow bubble formation.

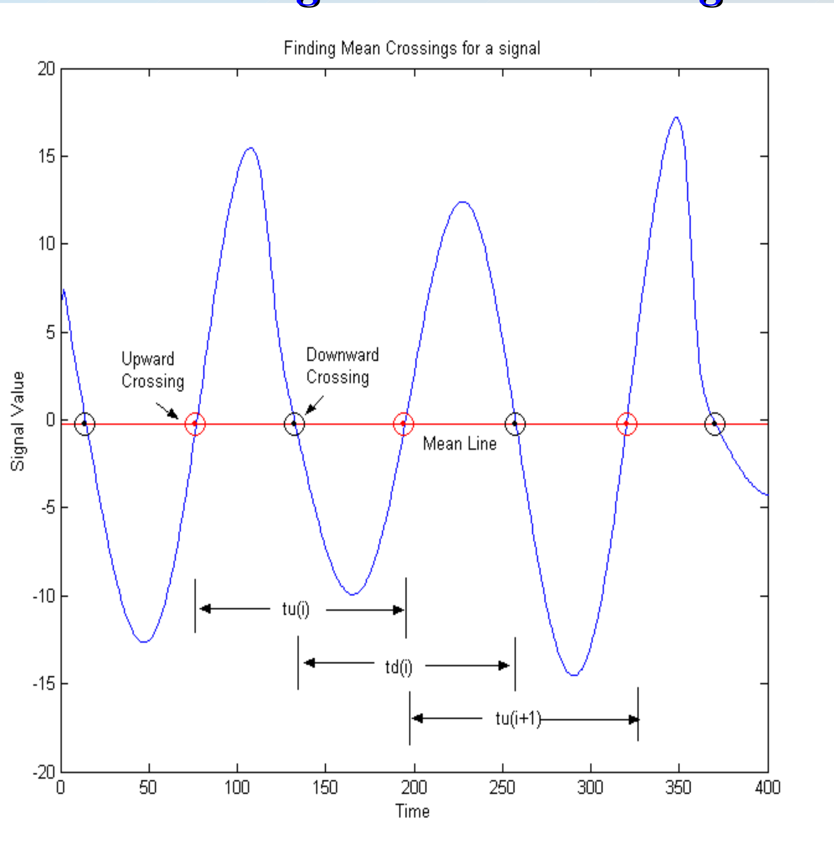
Return Maps(crossings-based) & period-doubling route to chaos

Differential Pressure

Return Maps



Finding Threshold Crossings



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Proposed Approaches

- Return maps characterize the system sufficiently well
- Developing methods based on return maps

TWO APPROACHES

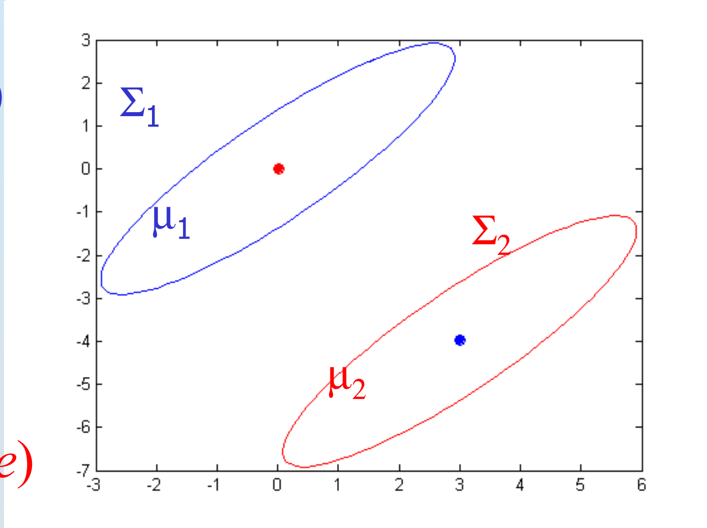
- Compare the joint PDFs of return maps
  - Compare means
  - Compare autocovariance matrices

- Reduce the joint probability distribution of points to fewer dimensions

Comparing the Means and Autocovariance matrices

Comparing the means

- Weighted or Mahalanobis Distance $(\mu_1-\mu_2)^T \Sigma^{-1} (\mu_1-\mu_2)$
- Use average Mahalanobis distance (based on two Σ matrices)



Comparing the autocovariance matrices (for structure)

$\Sigma_1=U_1\Lambda_1U_1^T$ with eigenvalues $\{\lambda_i\}$

$\Sigma_2=U_2\Lambda_2U_2^T$ with eigenvalues $\{\lambda_j\}$

- Covariance matrix defines the shape of the PDF (better for gaussian distribution)
- We propose a compound Statistic defined as:

$$\max\left(\frac{\Sigma\,\lambda_i\,\Pi\,\lambda_j}{\Sigma\,\lambda_j\,\Pi\,\lambda_i}\,,\;\frac{\Sigma\,\lambda_j\,\Pi\,\lambda_i}{\Sigma\,\lambda_i\,\Pi\,\lambda_j}\right)$$

To compare the structure of data points on the return map

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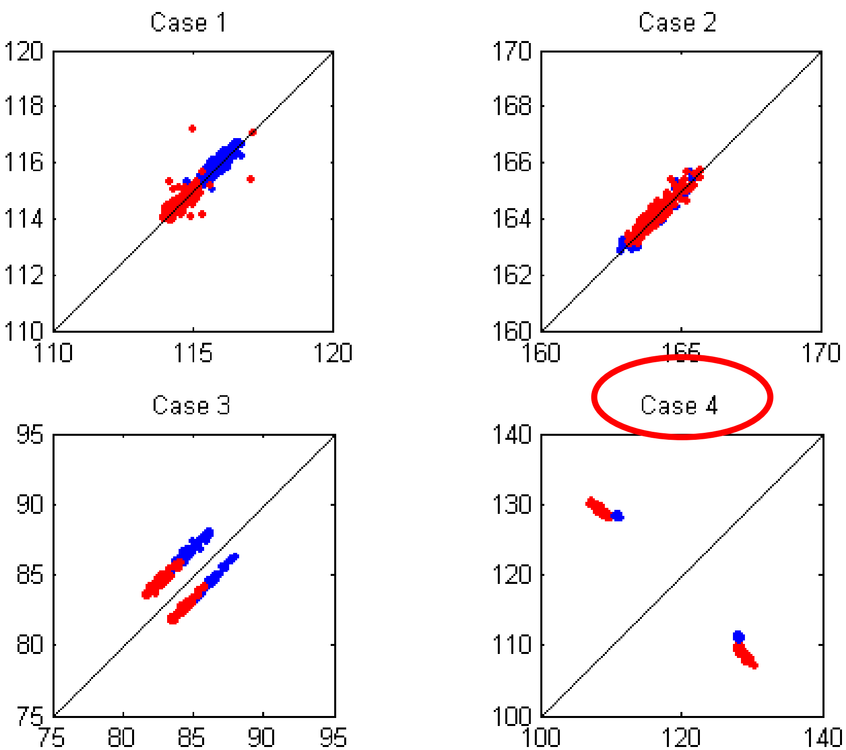
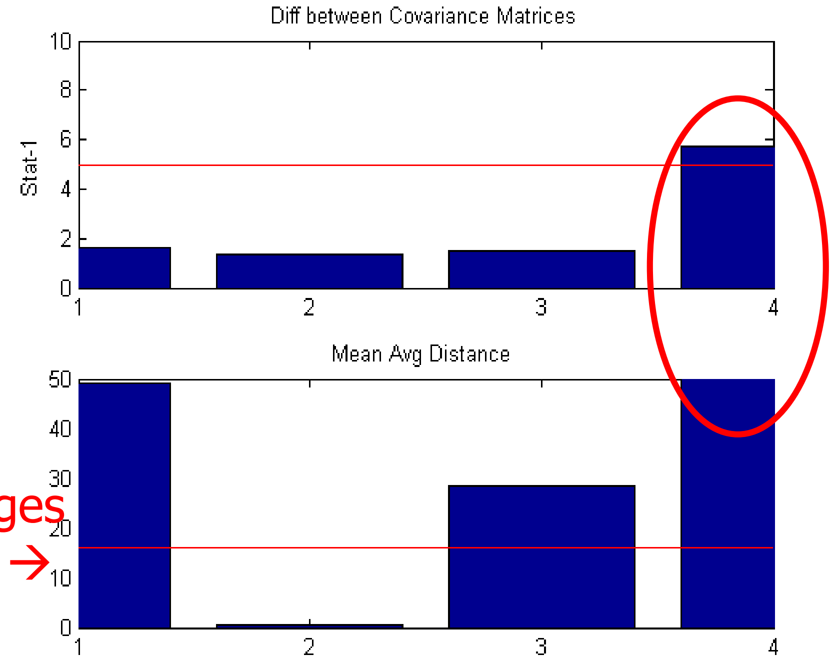
Example 1: Comparing Period-1 and Period-2 behavior



Blue dots represent the first one-third Of mean crossings, and red dots the last one-third. *Comparison provides information about the stationarity of the process*



The data points structure changes in case 4 as is shown in figure →

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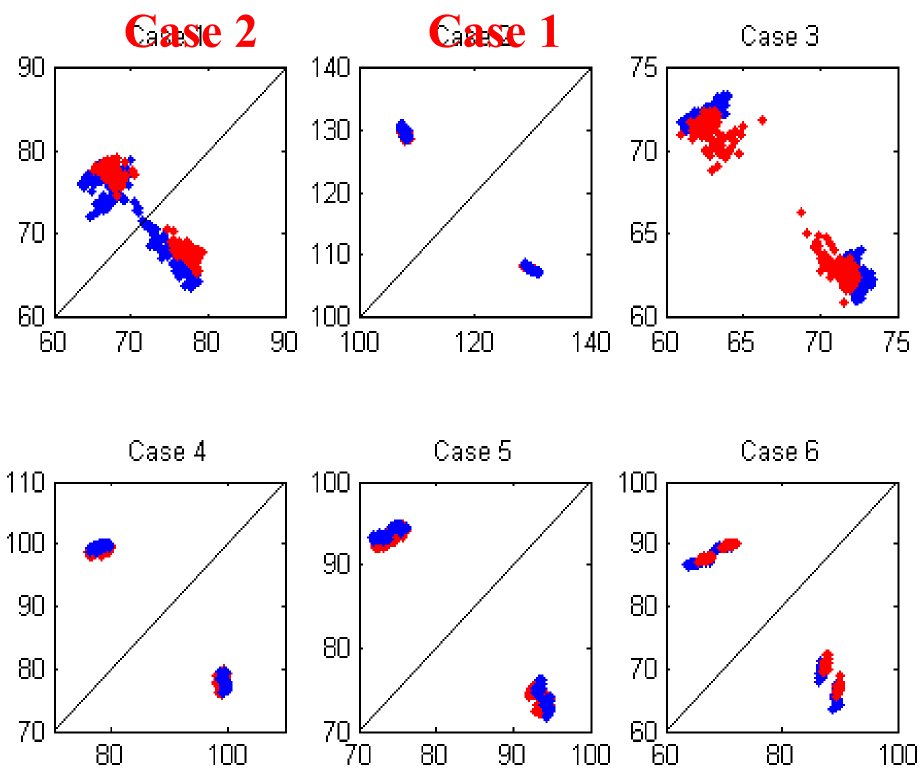
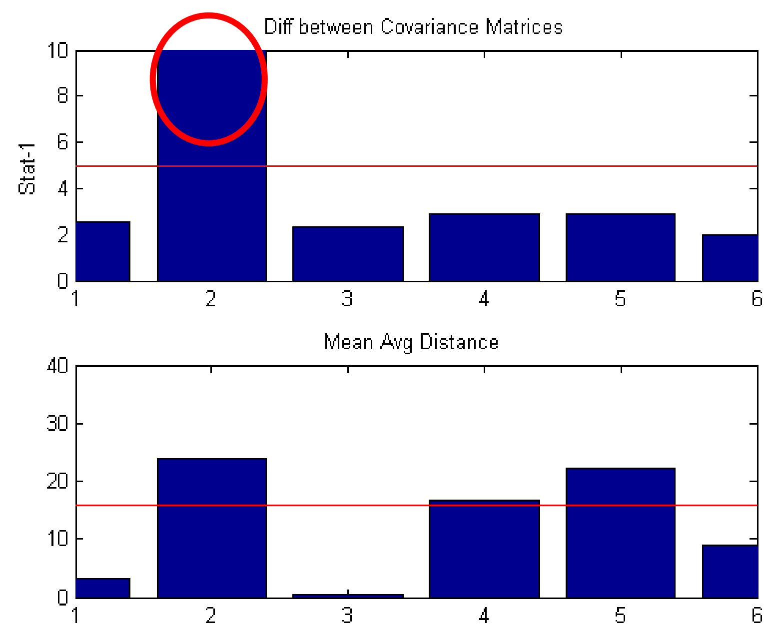
Example 2: Comparing chaotic States





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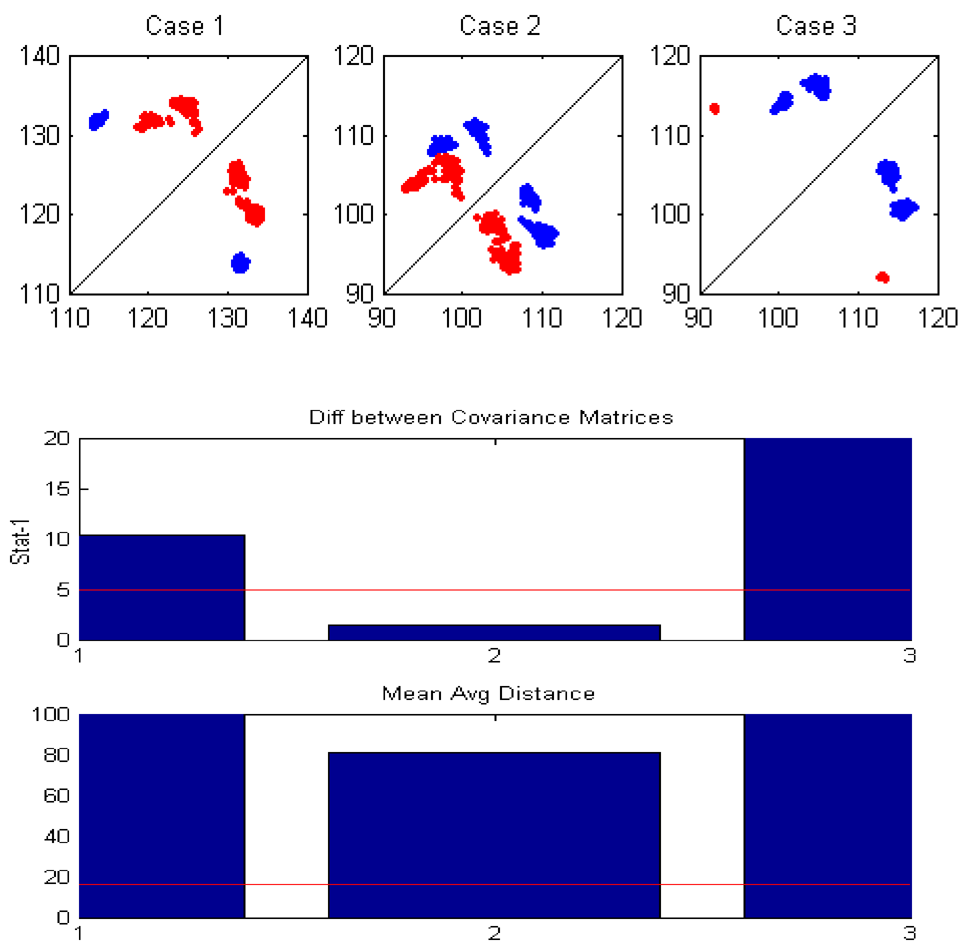
Example 3: Comparing Different Processes



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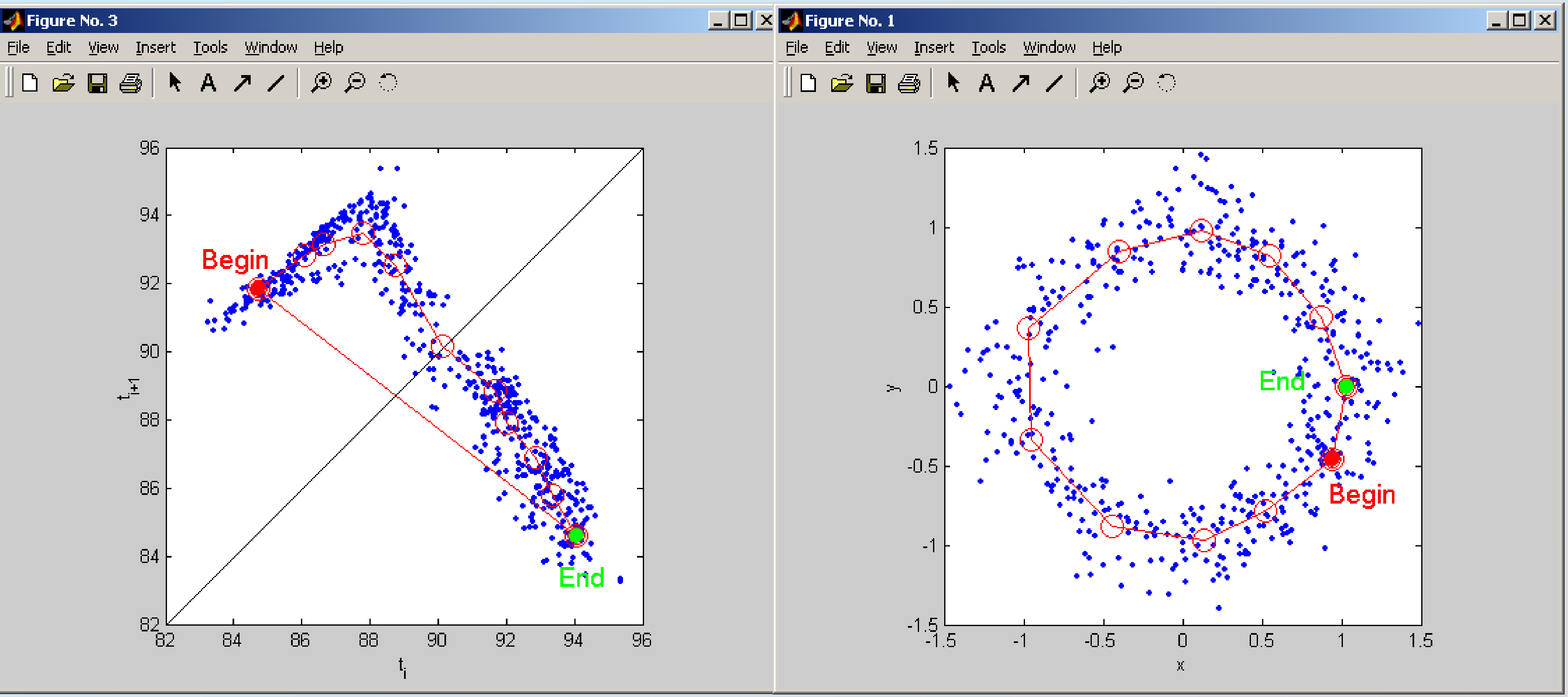
Principal Curves 1



Principal curves bend to the local density of data points and hence can approximate arbitrary structure in the data points

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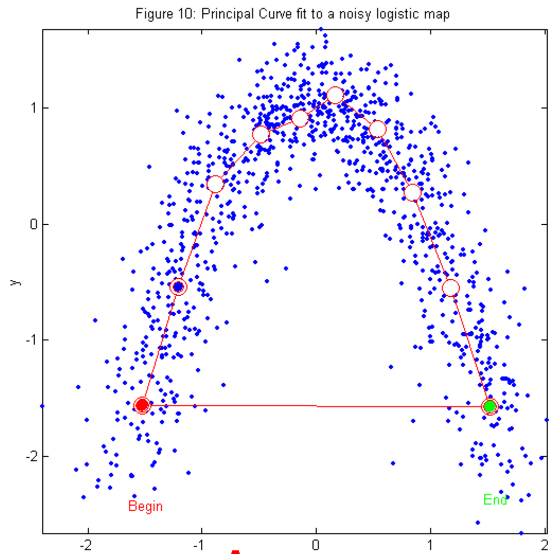
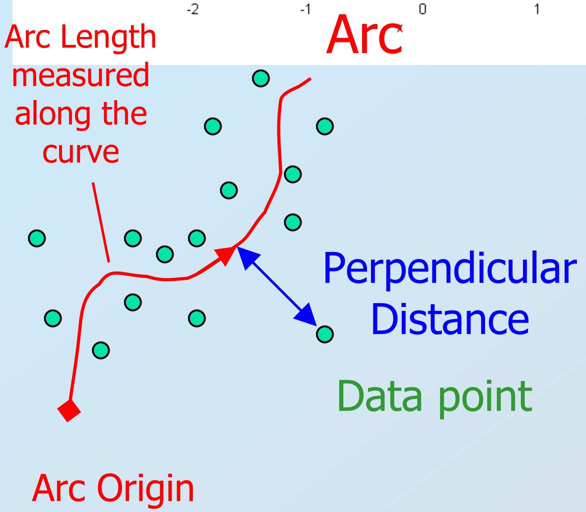
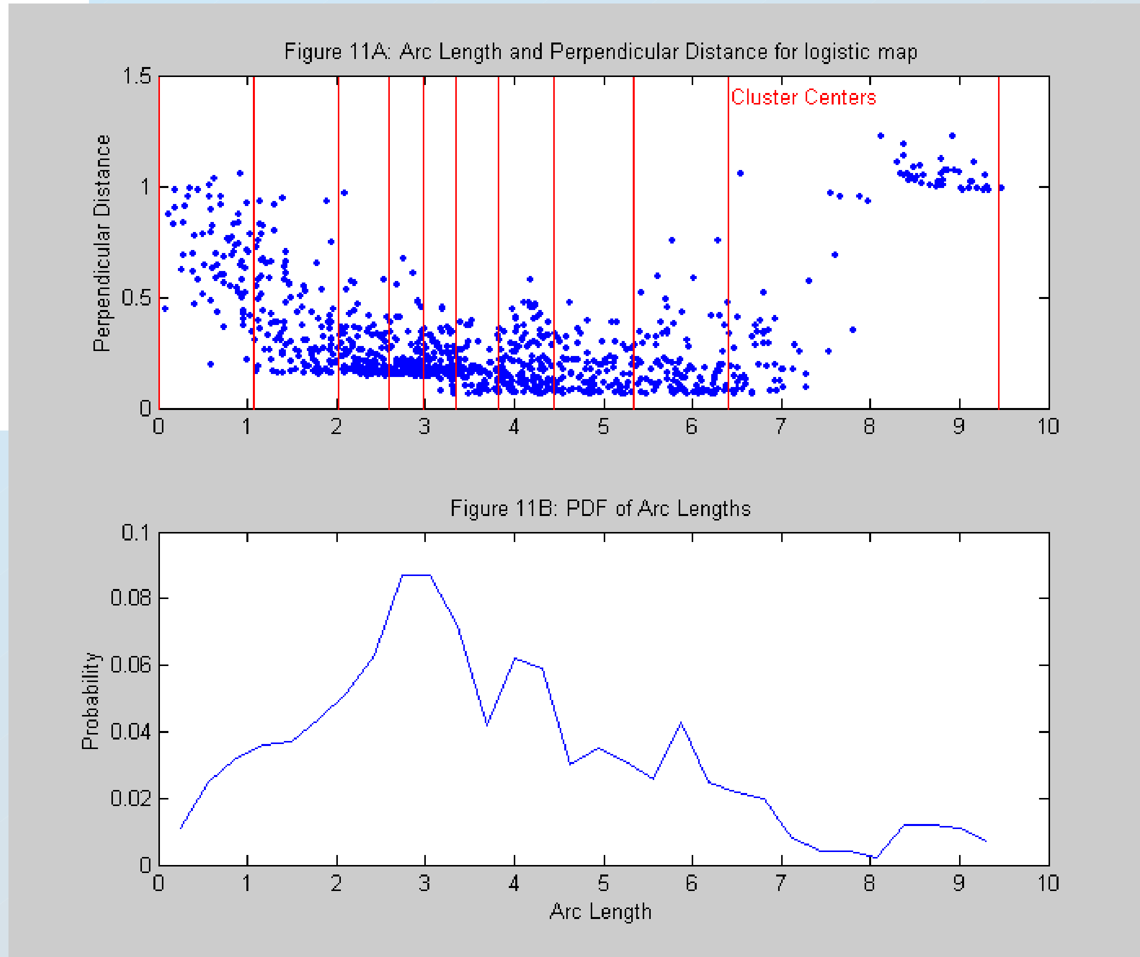
Principal Curves (2)







The distribution of data points can be expressed in the probability functions of its nonlinear principal scores or arc length along the principal curve

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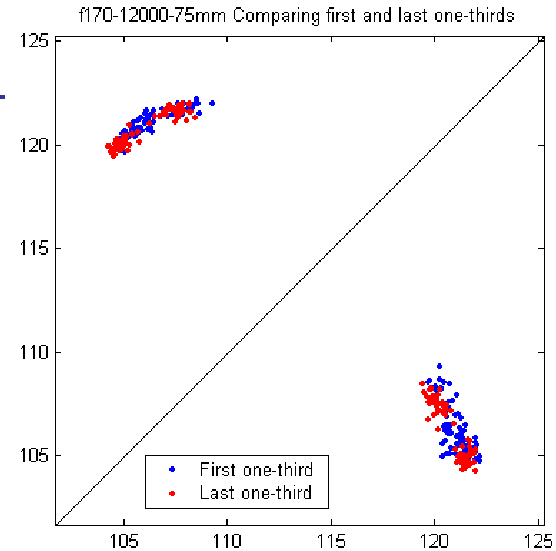
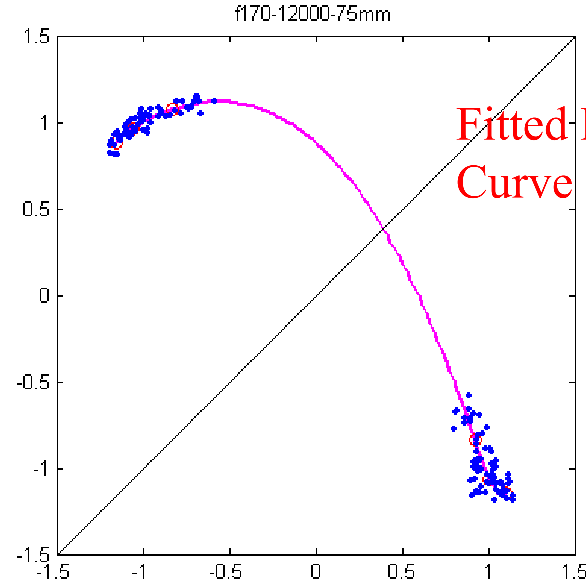
Putting NLPCA to work

- Fit a principal curve to one data set, and calculate
  - Arc lengths (characterize the *overall structure* of data points)
  - Perpendicular distances (residuals)
- For another data set, calculate
  - Arc lengths
  - Perpendicular distances
- Compare PDF of arc lengths
  - Chi-square statistic as a measure of dissimilarity
- Compare the sum of squared distances (SSD)
  - Measure of distance from the curve
- Can be used to
  - Gauge stationarity
  - Compare two processes
  - For fault diagnosis given sufficient *à priori* information

Example A: Testing non-stationarity

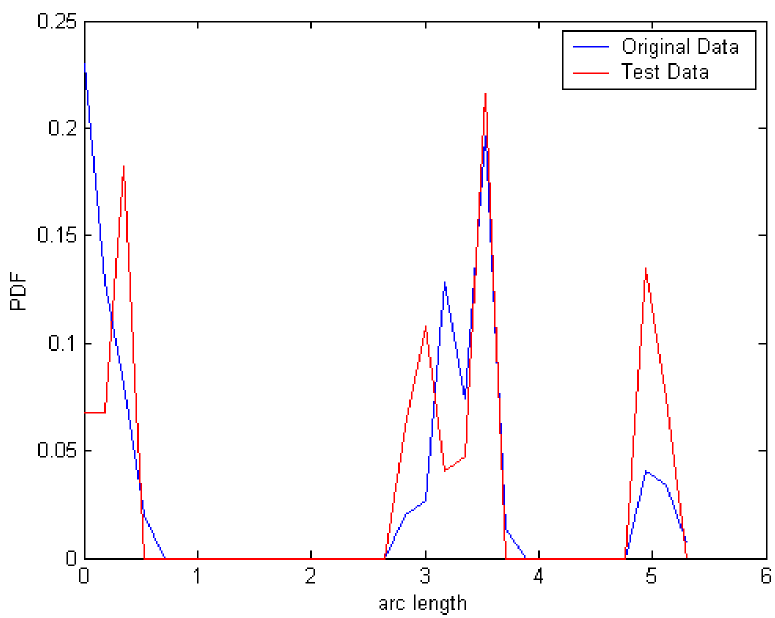
Overlaid first and last one-thirds





Fitted Principal Curve

Comparison of PDFs of arc lengths



Results

- Chi square value = 5.07
- Intermediate variability
- SSD on original data = 30.2
- SSD on test data = 65.4
- Can not reject the null

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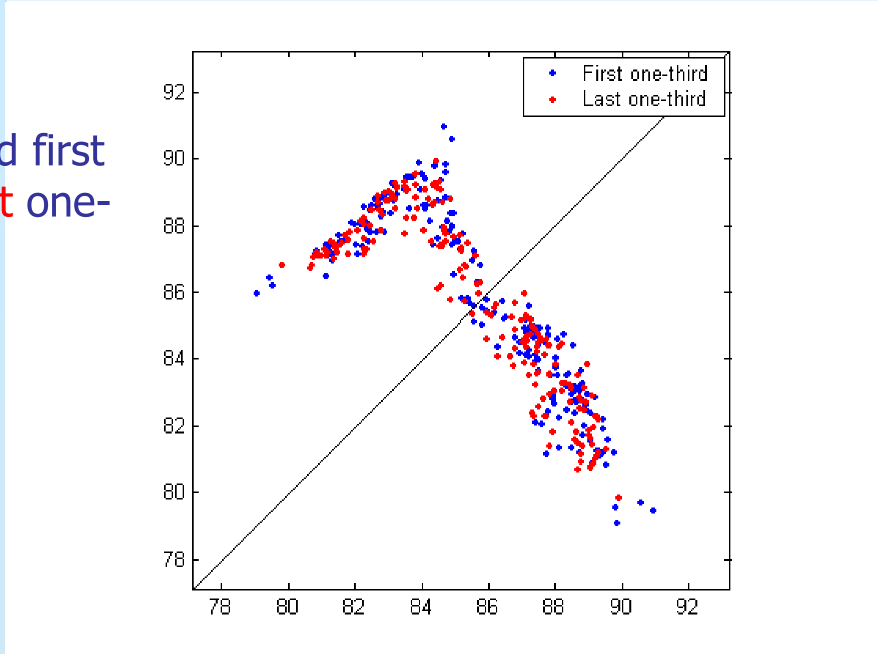
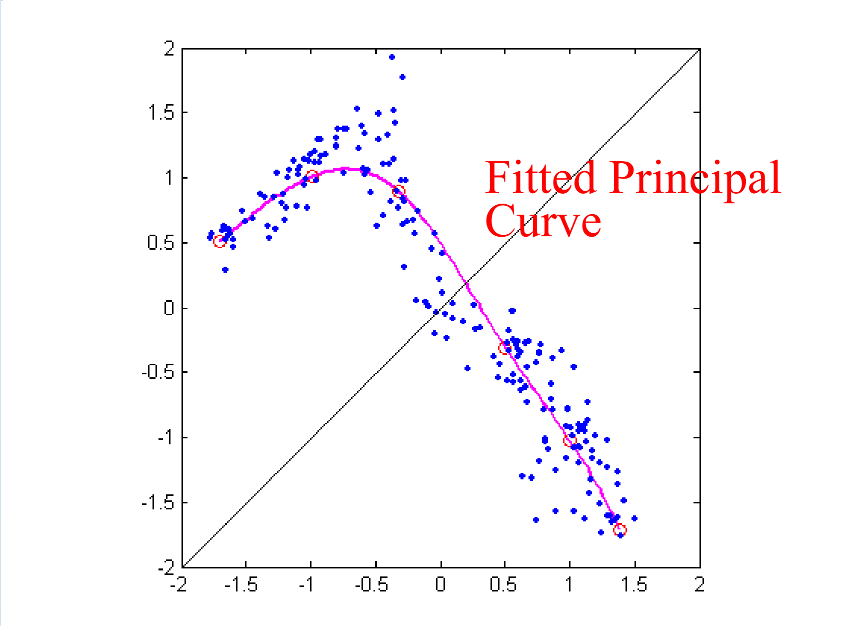
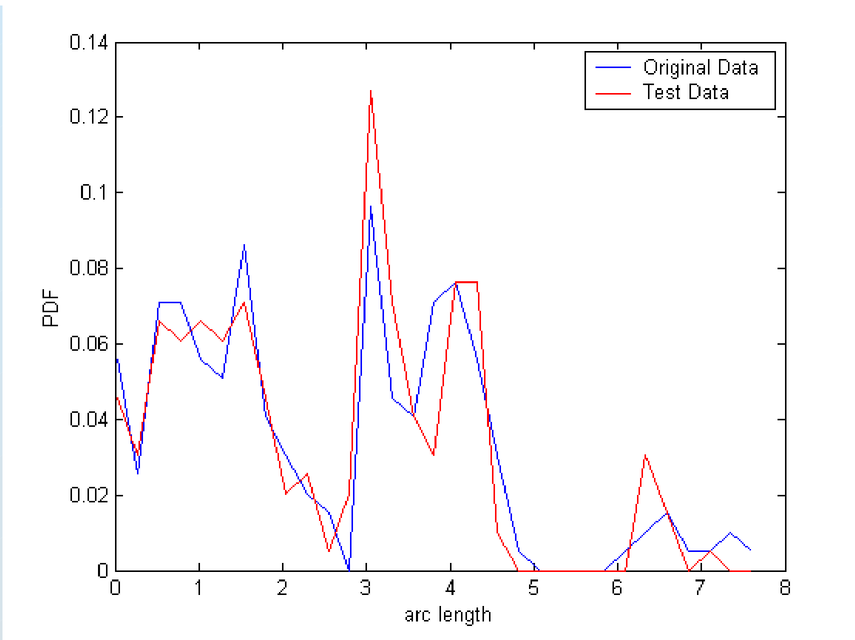
Example B: Testing non-stationarity

Overlaid first and last one-thirds





Comparison of PDFs of arc lengths



Results

- Chi square value = 0.80
- Very little variability
- SSD on original data = 56.0
- SSD on test data = 35.3
- Can not reject the null

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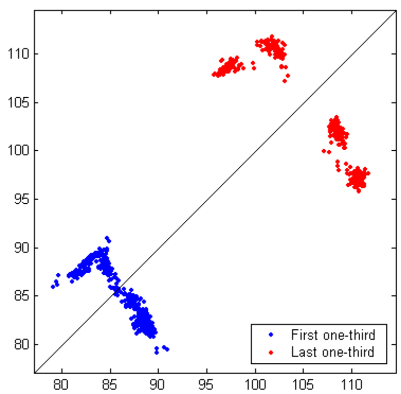
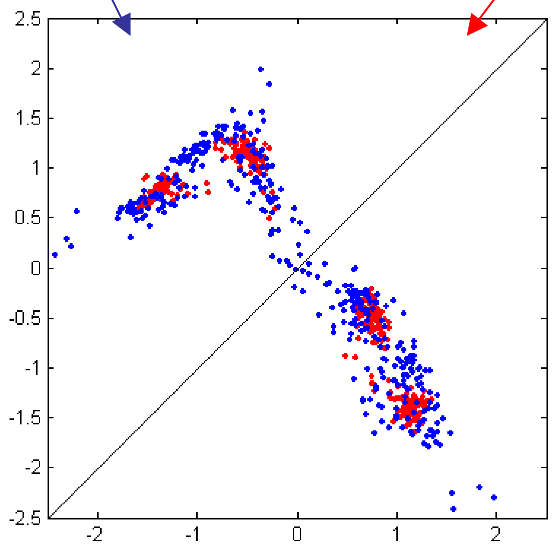
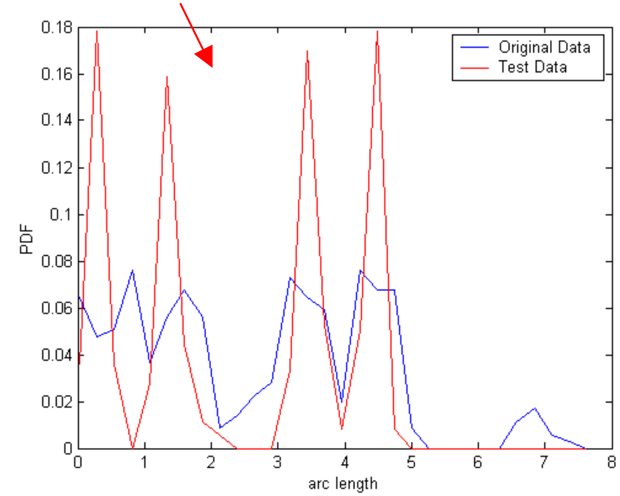
Example C: Comparing two processes

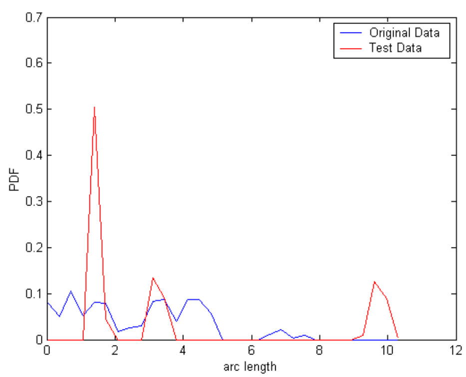


Overlaid return maps



Comparing the structure by scaling both processes separately





Results

- Chi square value = 45.8
- Large movement
- SSD on original data = 41.8
- SSD on test data = 34,074
- Reject the null at α=0.05

- Chi square value = 12.3
- Note four distinct peaks in the red PDF signifying period-4 behavior
- We used NLPCA to compare the structure of data points on the return maps

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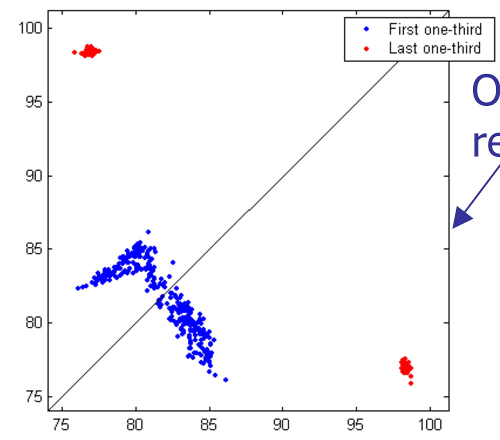
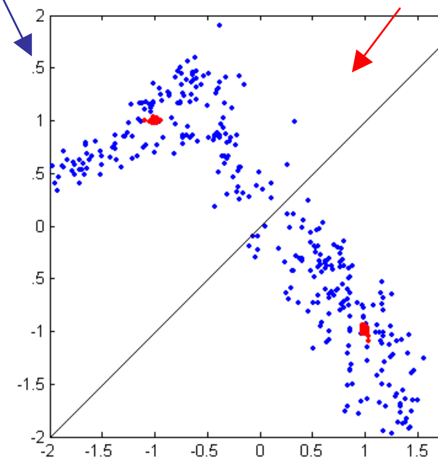
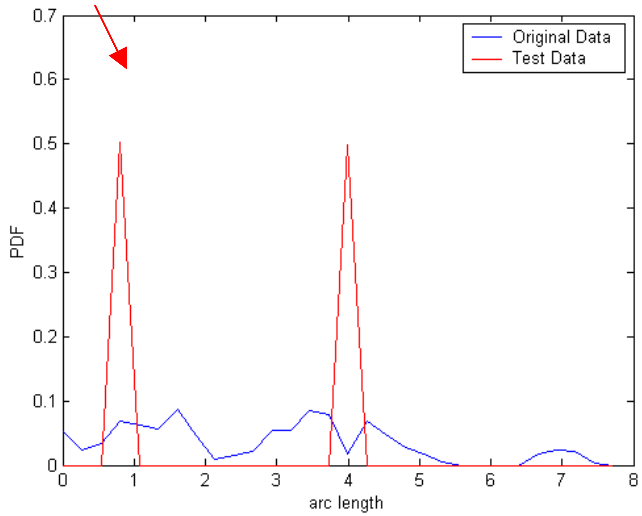
Example D: Comparing two processes

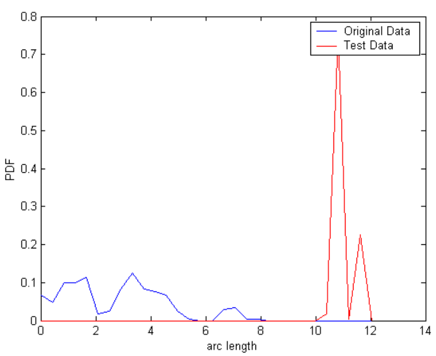


Overlaid return maps

Comparing the structure by scaling both processes separately







Results

- Chi square value = 68.9
- SSD on original data =123.0
- SSD on test data = 7332.5
- Reject the null at α=0.005

- Chi square value = 78.1
- Reject the null at α=0.005
- Note two distinct peaks in the red PDF
- Comparing chaotic state to period-2 behavior

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Conclusions

- General robust-to-noise methodology suggested for
  - Comparing two dynamic states
  - Can be utilized for
    - Testing stationarity
    - As a measure of [dis]similarity between dynamic states
    - System State detection
    - Comparing even chaotic states
    - Nonlinear Process Monitoring
- Demonstrated the ability to capture changes in structure
- Principal Curves very effective
- Can approximate attractor geometry by the principal curves
- Important implications for control and monitoring
- A generic approach which can be extended and adapted for other relevant tasks.