Measuring time irreversibility using symbolization

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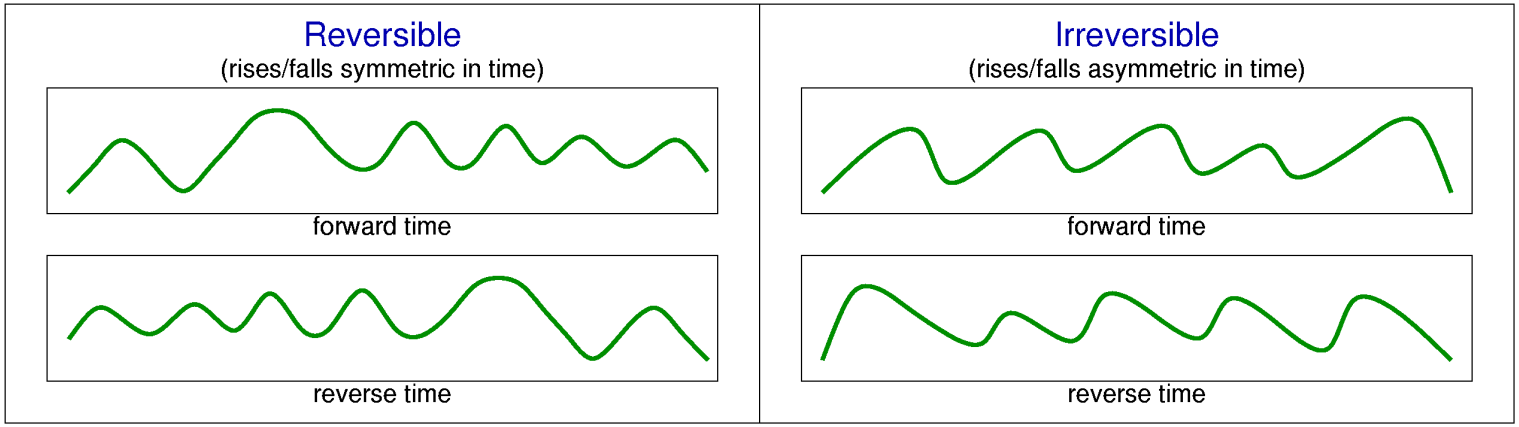
Abstract

Time irreversibility causes observed time series to look "different" when viewed in forward and reverse time (imagine playing an audio tape backwards). Linear Gaussian random processes (LGRP), and static transforms of LGRP, are inherently time reversible. Irreversibility is thus a “symptom” of nonlinear dynamics, and has gained much recent attention as a diagnostic for experimental data.

We present a test based on data symbolization that we find to be particularly suited for noisy, complex data. We illustrate our method with Hénon model data and experimental internal combustion engine measurements.

What is time irreversibility ?

Reversible time series “look similar” (eg, have similar dynamic flows) when viewed both in forward (natural) time or reverse time.

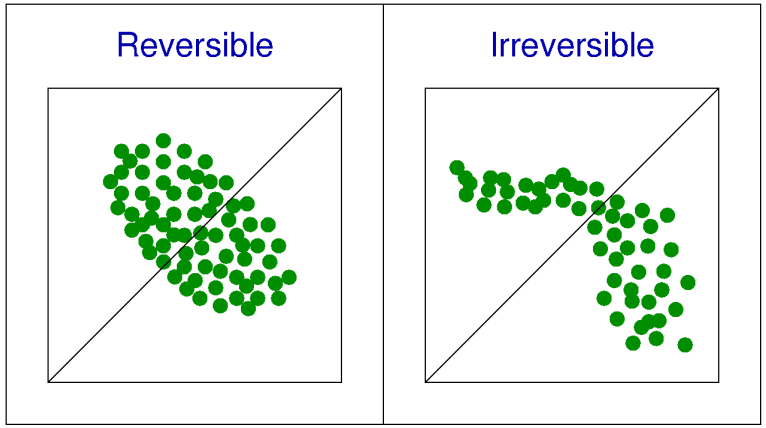


Linear Gaussian random processes (LGRP), as well as static transforms of LGRP, are reversible [Weiss]. For stationary data, verification of irreversibility for observed data excludes these types of processes as possible models.

Irreversibility is a “symptom” of nonlinearity [Cox].

Irreversibility affects temporally sensitive measures

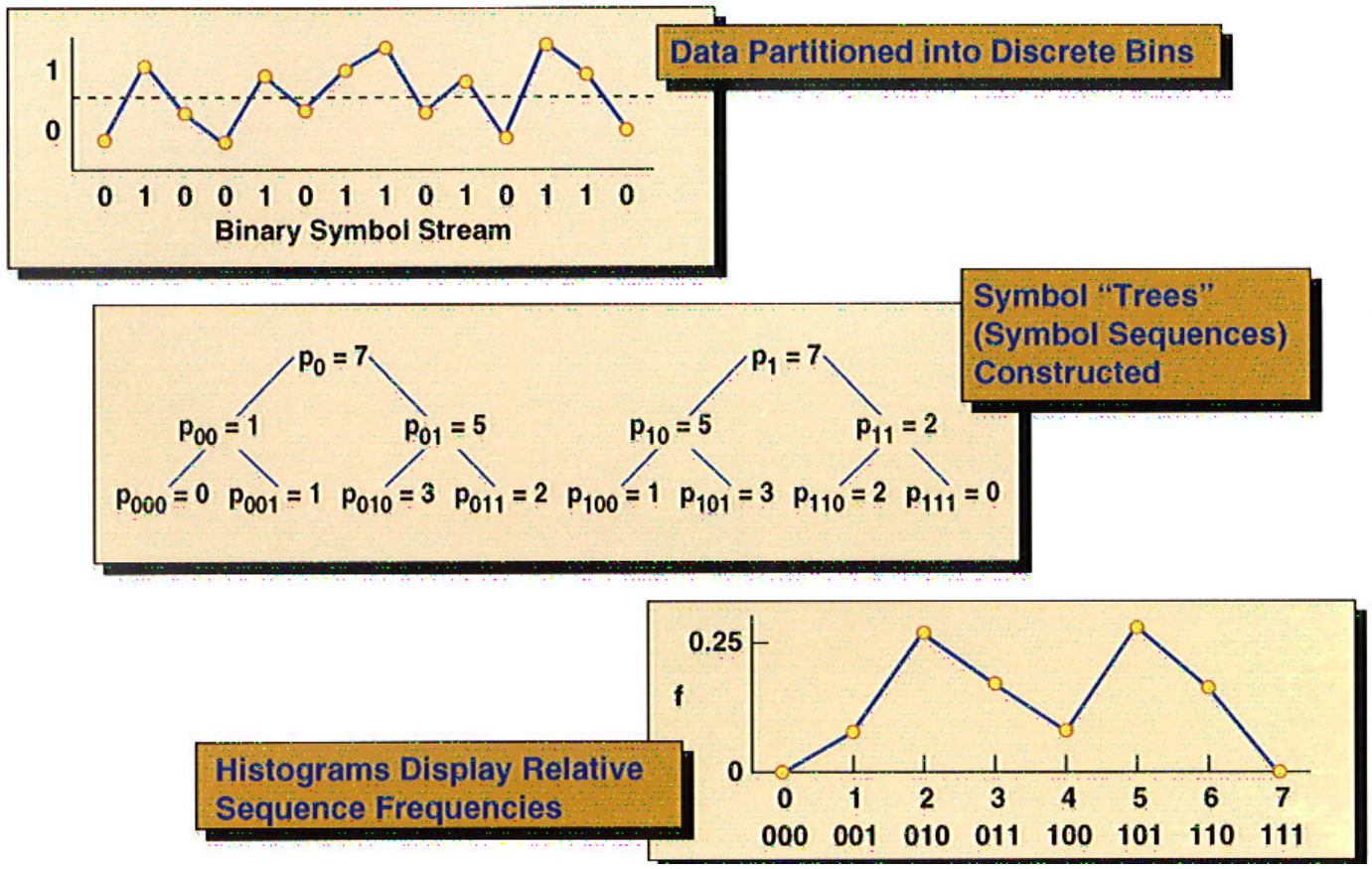
In low-dimensional return maps, irreversibility is readily manifest in asymmetry about the diagonal:



Higher-order cumulants (eg, moments about the mean) may also reflect temporal order. Second-order cumulants (eg autocorrelation) do not reveal irreversibility.

We develop an irreversibility metric using symbolization that is more general than either of the above.

What is symbolization ?



Symbol sequences with nonrandom frequencies reflect characteristic temporal patterns.

Not all sequences are equal

- Some sequences are intrinsically reversible:

  forward (1 0 1) = reverse (1 0 1)
- Other sequences are intrinsically irreversible:

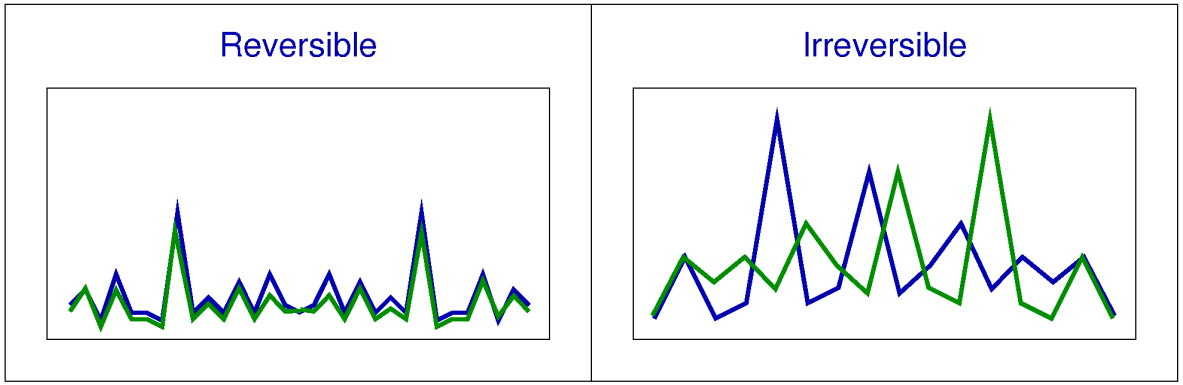
  forward (0 1 1) ≠ reverse (1 1 0)

  forward (2 1 0) ≠ reverse (0 1 2)
- The fraction of possible sequences that are irreversible depends on the selection of partition and sequence length

We specifically look for a statistically significant bias in forward and reverse occurrences of irreversible sequences as a measure of time irreversibility.

Simple comparative metrics of SSHs quantify irreversibility

Comparative metrics, such as the Euclidean norm or chi-square, allow quantification of differences between forward-time and reverse-time symbol sequences.



Many symbol sequences are temporally correlated.

Direct evaluation of significance in comparing full symbol-sequence histograms is problematic because of this correlation.

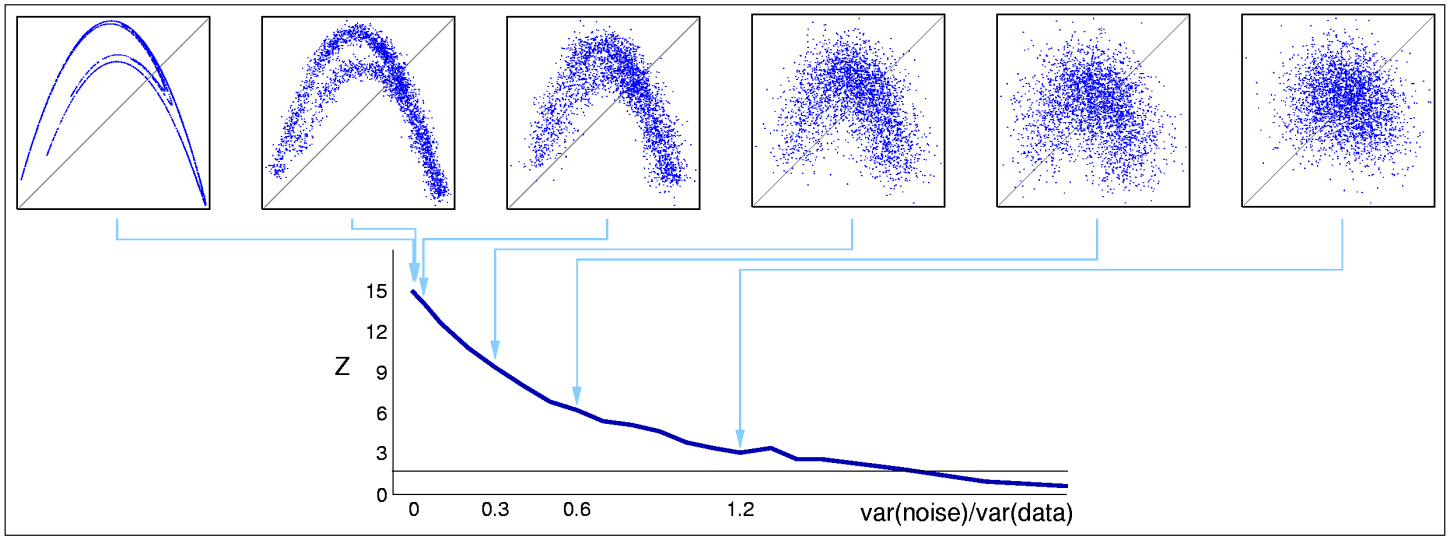
Targeted False Flipped Symbols:
a simple test with *a priori* significance (no surrogates required)

- Basic concept: Find a "target" sequence and compare the observed forward and reverse occurrences to the binomial likelihood for equal probability (H_o : p_{for}= p_{rev}= 1/2).
- Key steps:
  - **Find separate targets in both front and back halves of data.**
  - **For each target, compare observed forward and reverse frequencies in opposite half against H_o (corrected for temporal correlations - eg, remove occurrences < decorrelation interval).**
  - **Evaluate combined likelihood for both observed targets (approximated with *z* statistic for many observations).**

Use of target sequences increases resistance to noise and makes decorrelation much simpler.

Ex.1 : Hénon with additive noise

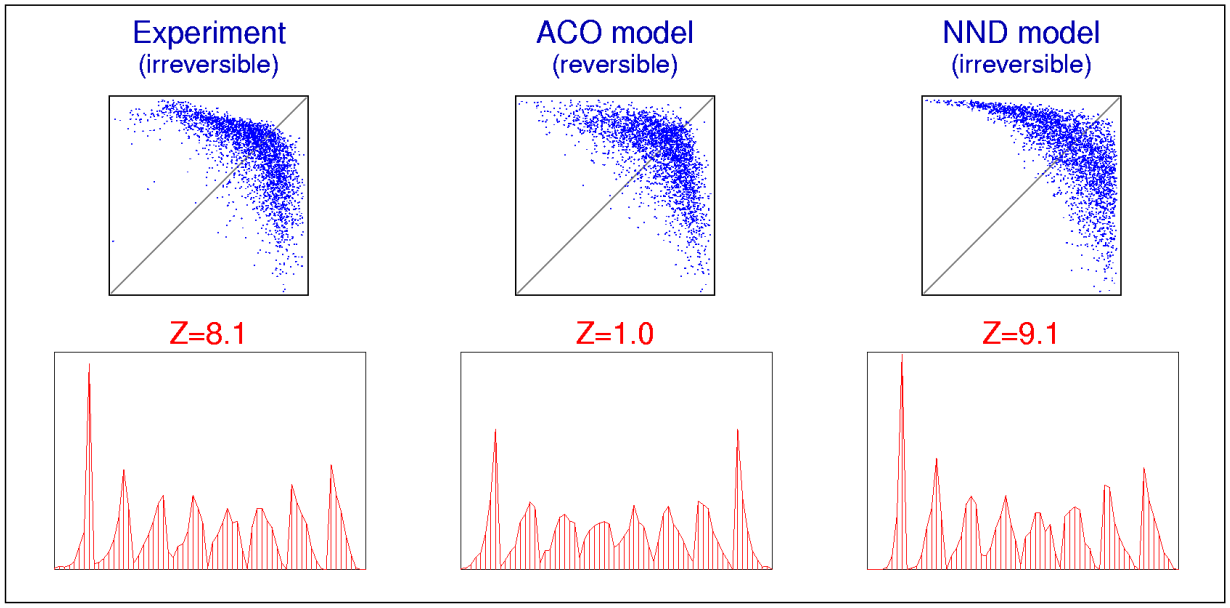
- 100 realizations of chaotic Hénon series (3000 records) with Gaussian additive noise
- TFFS test (z statistic) re-evaluated at varying noise levels (symbol parameters = 4 symbols, 3-symbol sequences)



The TFFS test is effective at high levels of additive noise and even more effective with dynamic noise.

Ex. 2 : Experimental & model internal combustion engine data

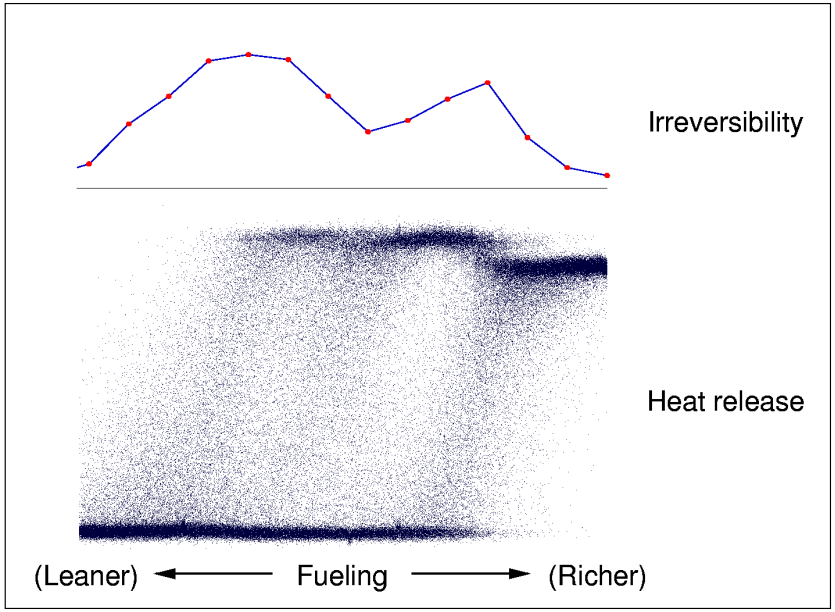
We use symbolization to distinguish alternative models for engine dynamics.



Irreversibility is important in showing which model better describes, or cannot describe, experimental dynamics.

Ex. 3 : Bifurcations in noisy experimental engine data

The nature of irreversibility changes around bifurcation points (data courtesy of R.M. Wagner).



Symbol-sequence analysis has real-world relevance for characterizing engine dynamics.

Summary

- Irreversibility in time series excludes linear Gaussian random processes, and static transforms thereof, as models.
- We have developed a simple test with *a priori* significance
- The TFFS test is resistant to noise because its basis is in symbolization
- We have used the test to discount a class of models for a physical process.

References

Time irreversibility

- Green JB Jr, Daw CS, Armfield JS, Finney CEA, Wagner RM, Drallmeir JA, Kennel MB, Durbetaki P (1999). "Time irreversibility and comparison of cyclic-variability models", SAE Paper No. 1999-01-0221.
- Stam CJ, Pijn JPM, Pritchard WS (1998). "Reliable detection of nonlinearity in experimental time series with strong periodic components", *Physica D* **112**, 361-380.
- Hoekstra BPT, Diks CGC, Allessie MA, DeGoede J (1997). "Nonlinear analysis of the pharmacological conversion of sustained atrial fibrillation in conscious goats by the class Ic drug cibenzoline", *Chaos* **7**, 430-446.
- Stone L, Landan G, May RM (1996). "Detecting Time's Arrow: a method for identifying nonlinearity and deterministic chaos in time-series data", *Proceedings of the Royal Society of London, Series B* **263**, 1509-1513.
- Heyden MJ van der, Diks C, Pijn JPM, Velis DN (1996). "Time reversibility of intracranial human EEG recordings in mesial temporal lobe epilepsy", *Physics Letters A* **216**, 283-288.
- Ramsey JB, Rothman P (1996). "Time irreversibility and business cycle asymmetry", *Journal of Money, Credit, and Banking* **28**, 1-21.
- Diks C, Houwelingen JC van, Takens F, DeGoede J (1995). "Reversibility as a criterion for discriminating time series", *Physics Letters A* **201**, 221-228.
- Timmer J, Ganert C, Deuschl G, Honerkamp J (1993). "Characteristics of hand tremor time series", *Biological Cybernetics* **70**, 75-80.
- Lawrance AJ (1991). "Directionality and reversibility in time series", *International Statistical Review* **59**, 67-79.
- Pomeau Y (1982). "Symétrie des fluctuations dans la renversement du temps", *Journal de Physique* **43**, 859-867.
- Cox DR (1981). "Statistical analysis of time series: some recent developments", *Scandinavian Journal of Statistics* **8**, 93-115.
- Weiss G (1975). "Time reversibility of linear stochastic processes", *Journal of Applied Probability* **12**, 831-836.

Symbolization

- Tang XZ, Tracy ER (1997). "Data compression and information retrieval via symbolization", *Chaos* **8**, 688-696.
- Finney CEA, Green JB Jr, Daw CS (1998). "Symbolic time-series analysis of engine combustion measurements", SAE Paper No. 980624.
- Lehrman M, Rechester AB, White RB (1997). "Symbolic analysis of chaotic signals and turbulent fluctuations", *Physical Review Letters* **78**, 54-57.
- Tang XZ, Tracy ER, Boozer AD, deBrauw A, Brown R (1995). "Symbol sequence statistics in noisy chaotic signal reconstruction", *Physical Review E* **51**, 3871-3889.
- Schwarz U, Benz AO, Kurths J, Witt A (1993). "Analysis of solar spike events by means of symbolic dynamics methods", *Astronomy and Astrophysics* **277**, 215-224.
- Crutchfield JP, Packard NH (1983). "Symbolic dynamics of noisy chaos", *Physica D* **7**, 201-223.

More references are available at http://www-chaos.engr.utk.edu/ReferenceLists.html

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