

# SYNCHRONIZATION OF COMBUSTION VARIATIONS IN A MULTI-CYLINDER SPARK IGNITION ENGINE

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## ABSTRACT

In this study, we experimentally investigated synchronization among combustion variations in different cylinders at fuel-lean conditions in an eight-cylinder spark ignition engine. Our results appear to confirm that synchronization can occur and that it becomes stronger as the overall equivalence ratio is reduced from stoichiometric. It also appears that the onset of synchronization is associated with bifurcation instabilities reported previously for combustion in single cylinders. We use both cross correlation and symbolic time series analysis to quantify the apparent relationships between pairs of cylinders and multi-cylinder groups. Extension of a simple dynamic model for single-cylinder combustion variations to the multi-cylinder case appears to agree with the observations and provides a basis for further studies. The occurrence of significant cylinder-to-cylinder synchronization may have significant implications for engine diagnostics and control.

## INTRODUCTION

Fuel-lean operation is important in spark ignition engines for reducing nitrogen oxides and hydrocarbon emissions as well as for improving fuel efficiency. One of the major practical constraints to implementing lean operation has been the onset of combustion instabilities as the lean limit is approached, resulting in an unacceptable level of misfires and partial burns. Thus it is becoming increasingly important to understand the fundamental causes of lean combustion instability and how the negative effects of this instability can be minimized.

Recent studies of cycle-to-cycle combustion variations within a single cylinder have shown that the behavior under lean fueling conditions is consistent with the onset of a noisy period-doubling bifurcation

sequence [1-5]. Such instabilities are a common characteristic feature of nonlinear dynamical systems in which future behavior is highly sensitive to small changes in past conditions. In the case of lean pre-mixed engine combustion, a likely mechanism for producing this instability would be the highly nonlinear effect of residual fuel and air left behind after each exhaust stroke on the subsequent ignition and flame propagation in succeeding combustion strokes. As has been previously demonstrated, the resulting dynamic behavior can be simulated using a simple mapping function that predicts the outcome of future combustion events (within a given cylinder) based on engine operating parameters and the outcome of past combustion events (in the same cylinder). The effects of higher-order phenomena such as in-cylinder fluid turbulence and vaporization of fuel droplets are represented as stochastic perturbations of engine parameters such as residual fraction and as-injected equivalence ratio. The purpose of such a hybrid low-order model is to capture the essential statistical features of the combustion instability without being computationally overwhelmed by details. Thus one sacrifices information about mixing and fluid turbulence in the cylinder for rapid prediction and the evaluation of large numbers (e.g., thousands) of cycles. This type of model is expected to be most useful for understanding general trends (e.g., for engine diagnostics) or for adaptive feed back control. A practical application of this kind of model can be seen in [6].

In this study we were specifically interested in determining if effects from lean combustion instability could extend beyond temporal coupling of each cylinder with itself to direct coupling between cylinders. Specifically, the presence of such coupling could lead to a dynamic condition known as synchronization, in which combustion events in one or more cylinders develop some type of predictable pattern relative to each other over time. Synchronization is a key characteristic feature of many multi-element nonlinear systems and has been reported for many types of physical systems [7-9]. Perhaps the most famous example is the case of the synchronizing clocks reported by Christiaan Huygens in 1673 [10]. In this case, Huygens demonstrated that clocks mounted on a common support were synchronized by the effect of small mechanical vibrations that were exchanged between them. Similar behavior has been reported for cellular flames [11]. If such synchronization occurs in engine combustion, it could have important consequences for engine design and control strategies.

Our approach included experimental combustion measurements from a multi-cylinder engine, analysis of the resulting data using new techniques from nonlinear dynamics theory, and comparison of our experimental observations with predictions from a multi-cylinder version of the low-order model for single-cylinder combustion variations. In the following sections we describe our experimental setup, data analysis procedures, and proposed model.

## **EXPERIMENTAL APPARATUS AND PROCEDURES**

The experiments discussed here were conducted on a 1999 4.6 liter, two valve V-8 Ford Expedition engine at five equivalence ratios. The nominal operating conditions for the experiments were 1200 rpm, 25 N-m brake torque, and 20° BTDC spark timing. The engine used production port fuel injection with a bore of 90.2 mm, a stroke of 90.0 mm, a compression ratio of 9.0, and a valve overlap of 27 degrees. Engine speed was controlled with an absorbing/motoring General Electric (GE) dynamometer.

In-cylinder pressure measurements recorded once per crank-angle degree were acquired from each cylinder at steady state conditions as equivalence ratio was varied from stoichiometric to very lean fueling conditions. The firing order for the cylinders was 1-3-7-2-6-5-4-8 (see Figure 1 for the cylinder layout). Feedback control was engaged to achieve an operating condition; once the condition was achieved, the feedback controllers were shut off, and the engine was run in open-loop mode. The only controller operating during the experiments was the GE dynamometer controller in speed mode, which actuated dynamometer torque to maintain engine speed. Since an absorbing/motoring dynamometer was used, constant engine speed was maintained even during very poor combustion at very lean fueling conditions. This strategy assured that the cycle-by-cycle engine combustion dynamics were minimally influenced by feedback control.

With the present data acquisition system, contiguous data sets of approximately 354 engine cycles (for all eight cylinders) can be collected at one time before the memory buffers are full. Repeat sets of contiguous data blocks can be collected after a 2 or 3 minute pause. For the results described below 3 contiguous data blocks were acquired at equivalence ratios of 1.00, 0.83, 0.71, 0.66, and 0.59. All experiments were then repeated a second time to evaluate reproducibility. Net heat release values for each cycle were calcu-

lated from the cylinder pressure data utilizing a variant of the Rassweiler and Withrow integration method [12].

## EXPERIMENTAL RESULTS

Our experiments clearly revealed the onset of combustion instabilities as fueling was leaned. Figures 2a and b illustrate typical observations for heat release patterns in all eight cylinders at stoichiometric and very lean fueling conditions. These plots depict pairs of sequential heat values for each cylinder and illustrate how each combustion event was related to its successor in time. We note that at near stoichiometric fueling (Fig. 2a), combustion variations were very small, producing a highly focused cluster of points around the average value. Detailed analysis of these small variations shows that they have a Gaussian distribution and appear to have occurred randomly in time.

At very lean fueling (Fig. 2b), the picture changed considerably. Here we observe a distinctive pattern of variations that is both far from Gaussian distributed and clearly not random. The detailed patterns vary from cylinder to cylinder, but all cylinders exhibited the characteristic “hooked” shape we have reported previously for single-cylinder studies [3, 5]. Our analyses for each individual cylinder also confirm that the statistical properties of the measurements are consistent with the noisy bifurcation model of Daw et al. [1]. However, for this multi-cylinder engine there is one complication; each cylinder appeared to progress through the bifurcation sequence at a different rate as fueling was leaned. Thus at each fueling condition, each cylinder was bifurcated to a greater or lesser extent and exhibited a distinctive pattern of cyclic combustion variations. We interpret this variation from cylinder to cylinder as resulting from nonuniformities in the distribution of fuel and air (especially) that cause differences in the as-injected equivalence ratio. Such nonuniformities are well known to occur and pose major challenges to engine designers [12].

Because we measured combustion in all eight cylinders simultaneously, we were able to investigate the possibility that combustion variations in different cylinders might have been related. We began by looking for cross correlations between cylinders as depicted in Fig. 3. Here we illustrate example behav-

ior over 1,000 engine cycles for selected cylinder pairs as fueling was leaned. Note that for one pair (F-0, F-5), a significant positive correlation developed when the as-injected equivalence ratio was adjusted to 0.66. Likewise, a significant anti-correlation appeared simultaneously for the cylinder pair (F-0,F-1), while there appeared to be no significant correlation for the pair (F-0,F-4). In general, we can summarize our observations from such cross correlations as follows:

- At stoichiometric fueling, we did not observe statistically significant cross correlations between any cylinder pairs. This indicates that combustion variations in each cylinder were effectively independent.
- As fueling was leaned, significant cross correlations apparently developed between combustion variations in some cylinder pairs. The cross correlations are in some cases positive and in some cases negative (anti-correlated).
- The distribution of positive and negative cross correlations among the different cylinder pairs was complex and did not appear to have any obvious relationship to cylinder location or firing order. We observed strong correlations between pairs fed by the same or opposite sides of the intake plenum, pairs on the same or opposing sides of the engine, and pairs close or far apart in firing order.
- The distribution of lean-fueling cross correlations apparently shifted over long periods of time (e.g., several hundred cycles). Over such periods, it appeared that strong correlations (or anti-correlations) developed between specific pairs, persisted for a significant interval (e.g., tens to hundreds of cycles) and then abruptly reduced in magnitude.
- At extremely lean fueling, combustion became very poor in all of the cylinders and the apparent degree of cross correlations between combustion variations diminished.

The lack of significant cross correlations at stoichiometric fueling implies that any potential coupling between cylinders was initially absent or significantly diminished. Likewise, the occurrence of apparently significant cylinder-to-cylinder correlations at lean fueling implies that some sort of coupling mechanism has been activated, allowing some of the cylinders to synchronize with each other. One might

conjecture, for example, that uncharacteristically large or small pressure oscillations are induced in the exhaust and intake manifolds when the combustion oscillations in individual cylinders become prominent (i.e., when partial or total misfires occur). Such changes in the pressure oscillations could impact the subsequent intake and exhaust strokes of other cylinders, thereby imposing additional variations in the effective in-cylinder equivalence ratio and stimulating the combustion to change. The apparent variations in bifurcation level from cylinder to cylinder would be expected to complicate the coupling interactions because the stronger oscillating cylinders would tend to “drive” those with weaker oscillations. While we did not measure the manifold pressure with sufficient accuracy to confirm this scenario in the current experiments, the impact of manifold pressure waves (acoustics) on engine combustion is a widely accepted phenomenon [12].

To further resolve the dynamic relationships between cylinders it is useful to apply more sophisticated analytical tools. One method we have found to be particularly useful is the symbol sequence analysis mentioned briefly in previous papers [2, 3]. Briefly, symbolization involves the transformation of an original time series of measurements into a sequence of discretized symbols. The advantage of such a transformation is that it is possible to readily detect and characterize the occurrence of nonrandom patterns, even when there is considerable experimental noise.

In the case of multi-cylinder combustion measurements, we are interested in observing nonrandom patterns both among the various cylinders (spatial variation) as well as over time. One useful symbolization consists of transforming the observed heat release values into a binary data stream (**0** or **1**) depending on where they fall relative to the median value. For each cylinder we use the median value for that cylinder so that biases in the nominal fueling levels are discounted. Once all the data have been symbolized, we evaluate interactions between specific cylinder pairs by constructing symbol “words” (i.e., short symbol sequences) according to the following definition:

$$S2(j) = [x(j-1), y(j-1), x(j), y(j), x(j+1), y(j+1)] \quad (1)$$

where  $S2(j)$  is the symbolic “state” of the cylinder pair at engine cycle  $j$ , and  $x(j)$  and  $y(j)$  are the sym-

bolic heat release values at cycle  $j$  for the first and second cylinder pair members, respectively. For convenience, we refer to each symbol sequence according to its index number which is defined by:

$$\text{Index}(S2) = x(j-1) \times 32 + y(j-1) \times 16 + x(j) \times 8 + y(j) \times 4 + x(j+1) \times 2 + y(j+1) \quad (2)$$

The full rationale for using the above indexing scheme is not important here, but it suffices to note that it allows us to uniquely refer to any particular pattern with a single number between 0 and 63.

Figure 4 illustrates how symbol sequence histograms can be used to depict the correlated and uncorrelated cylinder pairs shown previously in Fig. 3. On the abscissa we plot the sequence index number and on the ordinate we plot the relative frequency at which each possible sequence is observed. At stoichiometric fueling, all sequences are equally probable and the observed frequencies lie close to the heavy dashed line in Fig. 4. When combustion oscillations develop at lean fueling, peaks become prominent at sequence index values 12, 25, 38, and 51, corresponding respectively to the sequences **[0,0,1,1,0,0]**, **[0,1,1,0,0,1]**, **[1,0,0,1,1,0]**, and **[1,1,0,0,1,1]**. For uncorrelated behavior (F-0,F-4), the heights of these target peaks are all similar; that is, there is no favored relationship between the cylinder phases. Thus even when there are considerable oscillations in each cylinder, the lack of coupling means that we are just as likely to observe in-phase or out-of-phase behavior. When there is significant correlation (F-0,F-5), the relative heights of peaks 12 and 51 are higher than peaks 25 and 38, indicating that the combustion variations in both cylinders tend to remain in phase more frequently than out of phase. Peaks 25 and 38 are favored for the same reason when the cylinders become anti-correlated. In all cases, the mean height of all four peaks above background levels indicates the degree of joint bifurcation appearing in both cylinders. It is thus possible to use symbol sequence histograms to simultaneously evaluate both the degree of joint bifurcation and possible coupling.

Figures 5a and b illustrate how symbol “synchrograms” can be constructed to observe the evolution of joint cylinder bifurcation and correlation in time. Here we plot the observed sequence index against engine cycle number for example uncorrelated and anti-correlated cylinder pairs, respectively. For the uncorrelated pair, we observe that all four target peaks occur frequently, but these occurrences are spo-

radic and distributed relatively evenly over time. This pattern indicates that both cylinders are oscillating significantly, but that the oscillations are not related to each other in any consistent way. Conversely, for the anti-correlated pair, we observe significantly long uninterrupted periods (episodes) of relatively strong anti-correlated behavior (peaks 25 and 38). Such episodic behavior is highly suggestive of a noisy synchronization.

Episodic behavior among groups of cylinders larger than two can also be studied using a modified symbol sequence definition. Specifically, we define a 3-cylinder word as:

$$S3(j) = [x(j-1), y(j-1), z(j-1), x(j), y(j), z(j)] \quad (3)$$

where  $x(j)$ ,  $y(j)$ , and  $z(j)$  are symbolic heat release values for cylinders  $x$ ,  $y$ , and  $z$  measured in engine cycle  $j$ . The corresponding sequence index is

$$\text{Index}(S3) = x(j-1) \times 32 + y(j-1) \times 16 + z(j-1) \times 8 + x(j) \times 4 + y(j) \times 2 + z(j) \quad (4)$$

Figure 6 illustrates an example synchrogram result of applying the above modified sequence definition to two cylinder pairs with a common member. Note that in this case, there appears to be a clear episodic interaction among all three cylinders.

## PROPOSED MODEL FOR MULTI-CYLINDER SYNCHRONIZATION

Based on the above observations, we conjecture that a model for simulating the observed behavior could be constructed through revision and adaptation of the Daw et al. [1] model. Specifically, we suggest that the random noise term used for representing unmodeled engine perturbations should be modified to consist of a random component and a nonrandom component that reflects inter-cylinder coupling. From a dynamical systems perspective, this implies that multi-cylinder engines are more accurately described as noisy coupled map lattices, where each cylinder is a nonlinear mapping function that is coupled in varying degrees to the others. While the analysis of such complex dynamic structures is daunting, it would still seem reasonable to characterize different engines by the average degree of each coupling be-

tween different cylinders. Such a model might be useful in this regard (e.g., for data fitting). It is also conceivable that adaptive versions of this model (e.g., implemented as a neural network) might be used for on-board diagnostics and control.

To construct such a model, we iterated two replicates of the Daw et al. map, representing two different cylinders in a noisy bifurcated state. We further assumed that the dominant noisy parametric inputs to both cylinders were variations in the as-injected equivalence ratio. These equivalence ratio variations were assumed to be the sum of Gaussian variations (independent for each cylinder) and a small oscillating component that was either correlated or anti-correlated between the cylinders. The oscillating component was intended to represent the effect of manifold pressure waves that might effectively couple the cylinders. By varying the relative contribution of the oscillatory component, we studied the predicted effect of coupling. The level of total noise input to each cylinder was kept constant.

Example results of the above model are shown in synchrogram form in Fig. 7. An anti-correlated oscillating noise component was input at 5% (i.e., the percentage of noise variance contributed by the oscillating component). At these conditions, we observed apparent synchronization behavior that was very similar to some of our engine measurements for anti-correlated cylinder pairs. Thus it appears that the model predicts behavior that is at least qualitatively correct.

## **SUMMARY AND CONCLUSIONS**

Our experimental observations indicate the presence of significant cylinder-to-cylinder synchronization where the onset of synchronization corresponds to the development of a strong bifurcation in the cylinders. A simple mapping model with a shared noise component to reflect inter-cylinder coupling exhibited many of the characteristic features observed experimentally. Larger experimental data sets are needed to refine the picture of the possible cylinder-to-cylinder interaction patterns that can occur and how statistically significant they are. Future studies should address the physical mechanisms responsible for inter-cylinder coupling and how these phenomena should be modeled. Confirmation of the reported patterns here could have important implications for engine design and control.

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**FIGURE CAPTIONS**

- Figure 1 Cylinder and intake manifold configuration. Cylinder location is denoted by “C” for physical location and “F” for location in firing order.
- Figure 2 Heat release return maps for all eight cylinders at (a) stoichiometric and (b) very lean fueling conditions,  $\phi=0.66$ . Numbers in (a) refer to cylinder location.
- Figure 3 Cross correlation between selected cylinder pairs as a function of equivalence ratio. Significant positive and negative cross correlations occurred at an equivalence ratio of 0.66.
- Figure 4 Example symbol sequence histograms for selected cylinder pairs at very lean fueling conditions,  $\phi=0.66$ . Solid symbols and solid line correspond to cylinder pair (F-0,F-5) which had a significant positive correlation. Hollow symbols and broken line correspond to pair (F-0,F-4) which had no significant correlation. The heavy dashed line represents the case where all sequences are equally probable which was approached at stoichiometric fueling.
- Figure 5 Symbol synchrograms for (a) uncorrelated and (b) anti-correlated cylinder pairs at very lean fueling conditions,  $\phi=0.66$ . Dark bands of repeated symbol sequences reveal combustion oscillations. A persistent bias in these bands indicates apparent synchronization.
- Figure 6 Symbol synchrogram for three cylinders (F-0,F-4,F-6) at very lean fueling conditions,  $\phi=0.66$ . Here we observe a persistent bias in the relationship of the three cylinders as a group.
- Figure 7 Symbol synchrogram for anti-correlated model simulation. Here persistent bands reveal the anti-correlated coupling deliberately added to the model. The observed patterns are qualitatively similar to the experimental observations for anti-correlated cylinder pairs.

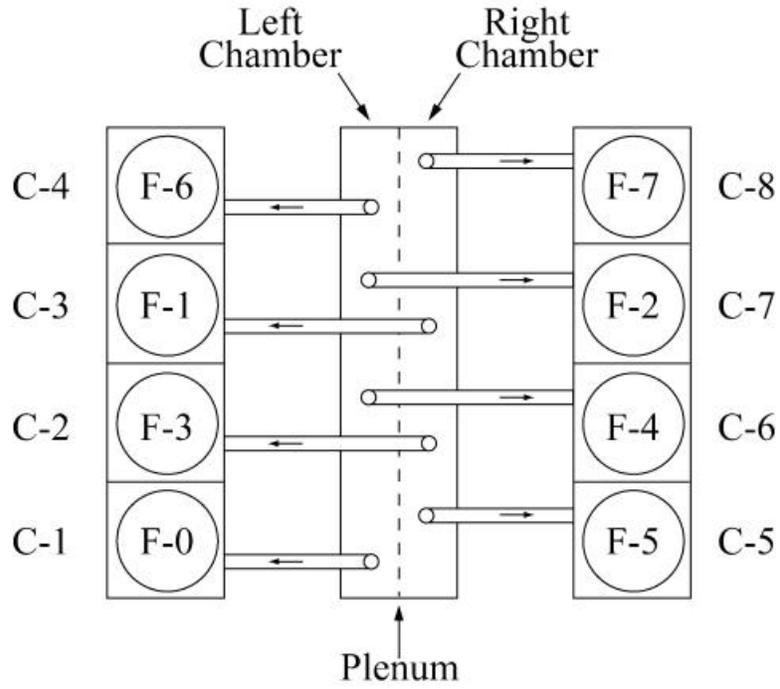
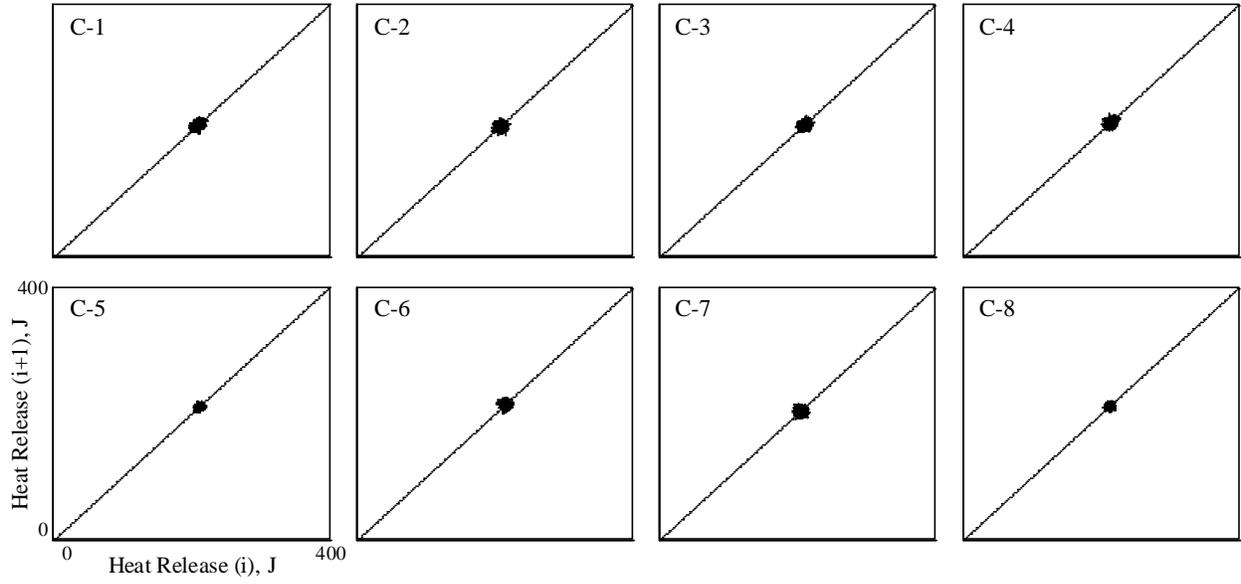
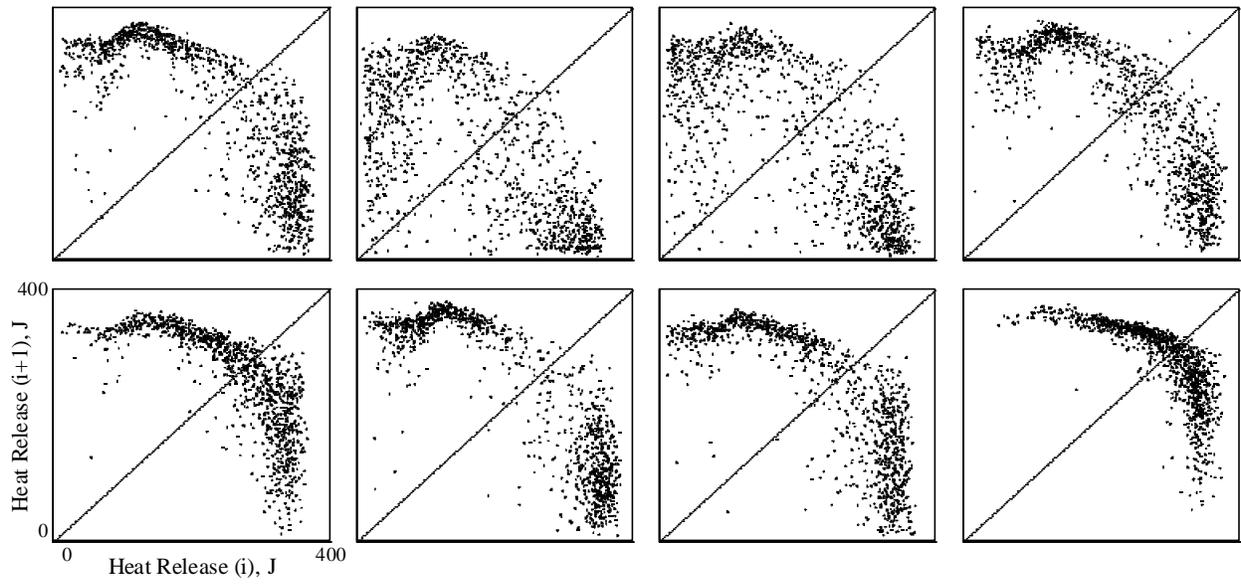


Fig. 1 Cylinder and intake manifold configuration. Cylinder location is denoted by “C” for physical location and “F” for location in firing order.



(a)



(b)

Fig. 2 Heat release return maps for all eight cylinders at (a) stoichiometric and (b) very lean fueling conditions,  $\phi=0.66$ . Numbers in (a) refer to cylinder location.

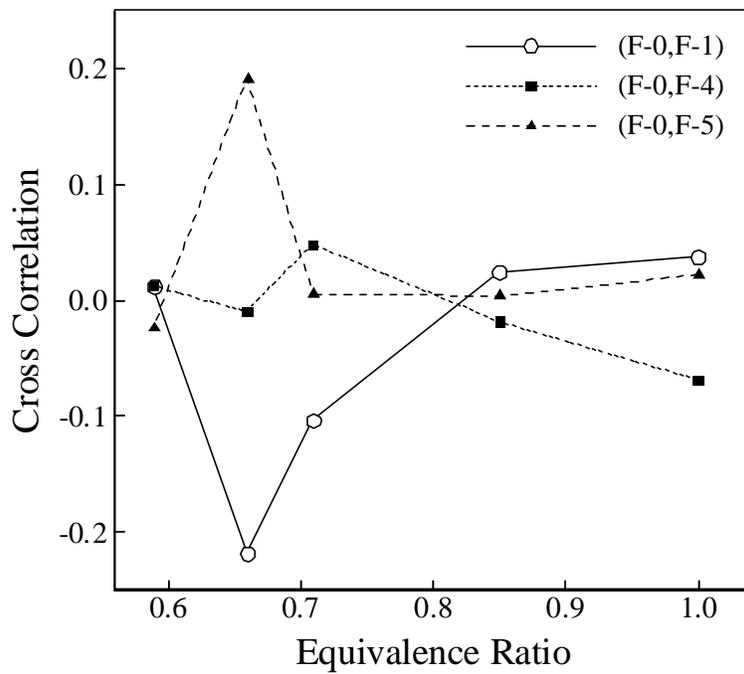


Fig. 3 Cross correlation between selected cylinder pairs as a function of equivalence ratio. Significant positive and negative cross correlations occurred at an equivalence ratio of 0.66.

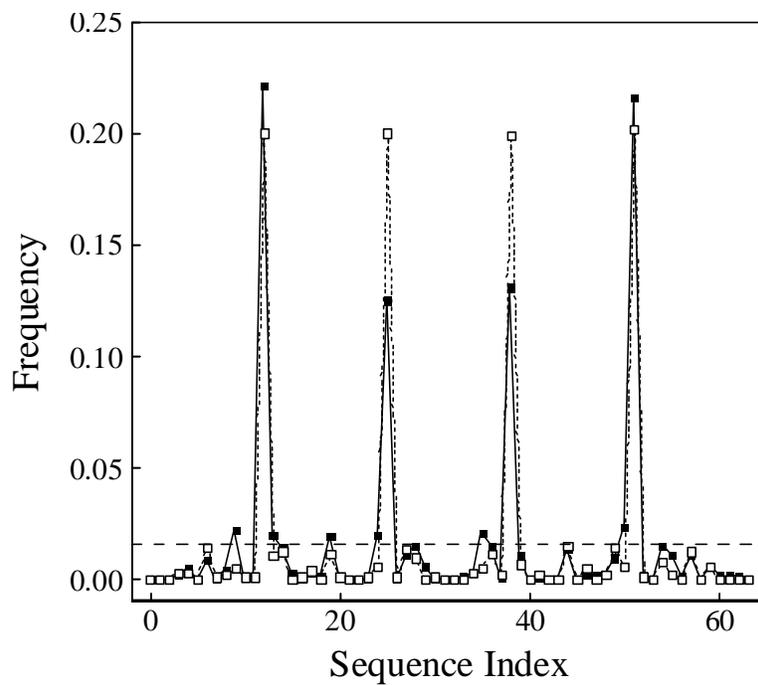
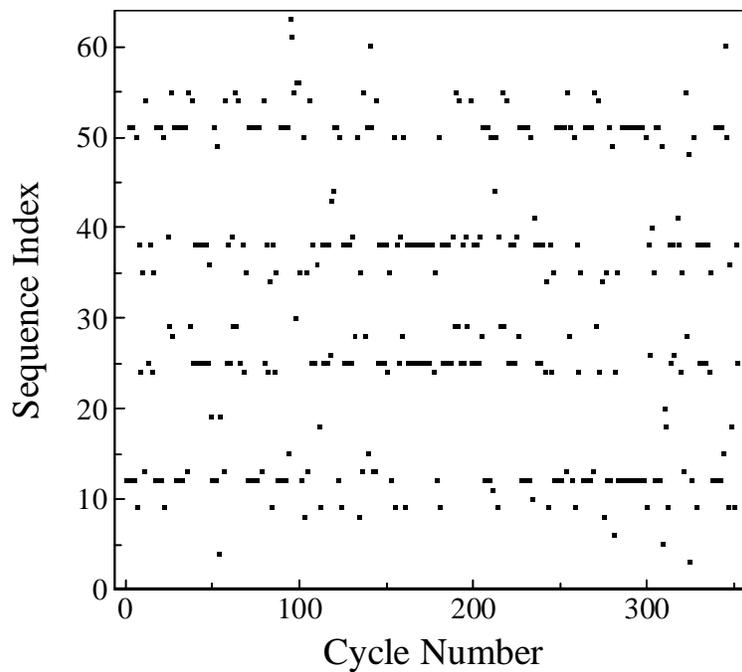
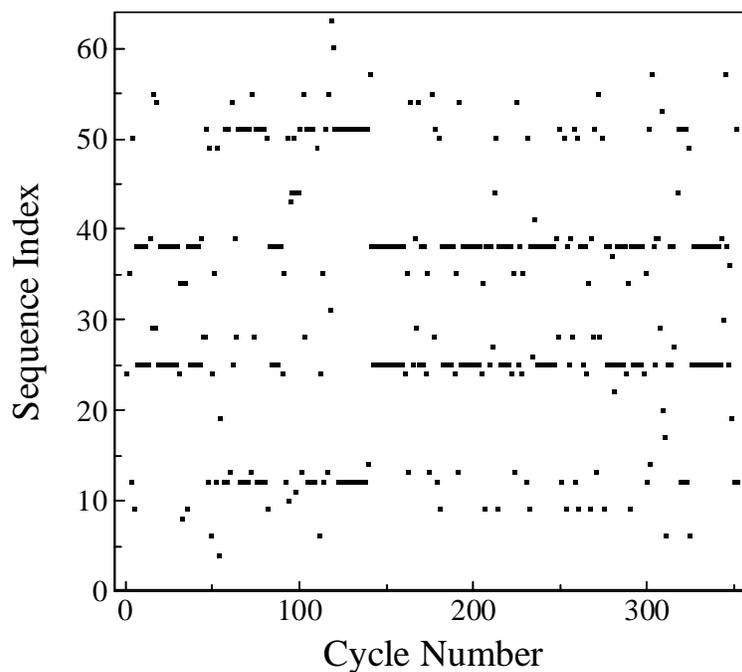


Fig. 4 Example symbol sequence histograms for selected cylinder pairs at very lean fueling conditions,  $\phi=0.66$ . Solid symbols and solid line correspond to cylinder pair (F-0,F-5) which had a significant positive correlation. Hollow symbols and broken line correspond to pair (F-0,F-4) which had no significant correlation. The heavy dashed line represents the case where all sequences are equally probable which was approached at stoichiometric fueling.



(a)



(b)

Fig. 5 Symbol synchrograms for (a) uncorrelated and (b) anti-correlated cylinder pairs at very lean fueling conditions,  $\phi=0.66$ . Dark bands of repeated symbol sequences reveal combustion oscillations. A persistent bias in these bands indicates apparent synchronization.

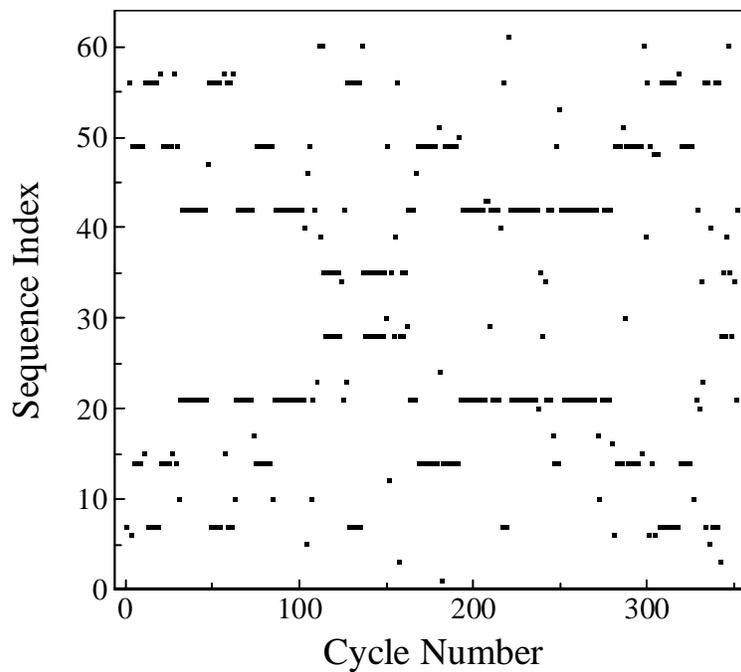


Fig. 6 Symbol synchrogram for three cylinders (F-0,F-4,F-6) at very lean fueling conditions,  $\phi=0.66$ . Here we observe a persistent bias in the relationship of the three cylinders as a group.

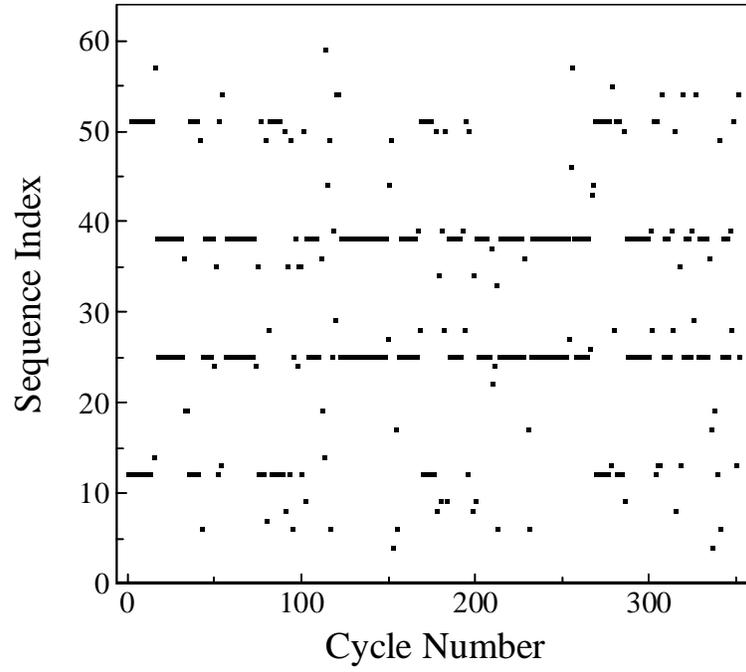


Fig. 7 Symbol synchrogram for anti-correlated model simulation. Here persistent bands reveal the anti-correlated coupling deliberately added to the model. The observed patterns are qualitatively similar to the experimental observations for anti-correlated cylinder pairs.