Enhancing the operation of a pulsed combustor with trajectory-correction control

K.D. Edwards, K. Nguyen*
University of Tennessee
Knoxville TN 37996-2210

C.S. Daw
Oak Ridge National Laboratory
Oak Ridge TN 37831-8088

Abstract

The behavior of a laboratory-scale pulsed combustor is shown to undergo a global bifurcation as the equivalence ratio of the propane-air mixture is reduced toward the lean flammability limit. At near-stoichiometric and moderately lean conditions, the acoustically driven combustion oscillations exhibit small, stochastic cycle-to-cycle variations. At very lean conditions, the available fuel inventory is rapidly consumed and slowly restocked resulting in a high degree of variability in combustion quality over the span of several acoustic cycles. This mode of operation is very unstable and often leads to misfire and flameout of the combustor. A feedback control strategy is applied which occasionally applies small, appropriately timed injections of supplemental fuel to hasten the restocking process and push the trajectory of the system toward the more stable operating mode. Emission levels of unburned hydrocarbons and NO_x are monitored to demonstrate the effectiveness of feedback control. Possible improvements for the control strategy are recommended.

Motivation

Pulsed combustors are known to have significantly higher convective heat and mass transfer rates and produce lower pollutant emission levels than steady-flow combustors operating at similar conditions (Hanby 1969; Dec and Keller 1989; Arpaci *et al.* 1991; Gemmen *et al.* 1991; Keller and Hongo 1990; Keller *et al.* 1994). To make full use of the benefits of pulsed combustion, it would be desirable to operate the combustor at lean conditions to increase the fuel efficiency and further reduce pollutant emission levels. Unfortunately, recent studies have shown that, like many other combustion systems, pulsed combustors become unstable at lean conditions, leading to misfire and, eventually, unrecoverable flameout. The instabilities often develop well above the lean flammability limit ($\phi \approx 0.54$ for propane-air) (Edwards *et al.* 1998; Edwards *et al.* 2000; Edwards 2000). Furthermore, the dynamic behavior of pulsed combustors has been shown to be highly nonlinear, especially at lean conditions (Sterling and Zukoski 1991; Daw *et al.* 1992; Sterling 1993; Daw *et al.* 1994; Daw *et al.* 1995; Margolis 1994; Edwards *et al.* 1998). Daw *et al.* (1992) suggested that, due to the nonlinear nature of pulsed combustion, linear control algorithms would prove less effective in dampening the combustion instabilities and extending the practical operating regime of pulsed combustors than would control strategies based upon nonlinear dynamics. In studies by Rhode *et al.* and In *et al.*, various nonlinear control techniques were successfully applied to an analytical pulsed-combustor model developed by Richards *et al.* (1991) at the former Morgantown Energy Technology Center (METC) — currently the National Energy Technology Laboratory (NETL), in part — which has been shown to predict nonlinear and deterministically chaotic behavior at certain operating conditions (Daw *et al.* 1992; Daw *et al.* 1995).

Rhode *et al.* (1995) applied a variety of control techniques to the METC model to eliminate the pressure oscillations or to promote regular oscillations in order to delay the onset of chaos. The friction factor of the tailpipe was used as the control parameter in this study. A derivative control scheme was applied in which the friction factor was increased by adding to it a value proportional to the derivative of temperature with respect to time. The derivative control scheme was found to be effective at eliminating the pressure oscillations and forcing the model to operate in a steady mode. Adaptive map-based control schemes were found to be effective in controlling the oscillatory behavior. Using occasional and recursive proportional feedback, the map-based control scheme was able to prevent the model from going through the period-doubling route to chaos described by Daw *et al.* (1995) and avoid flameout, thus extending the

*Corresponding author

pulsed operating regime of the model to include shorter flow times.

In *et al.* (1997) sought to delay flameout of the model by maintaining the chaotic pressure oscillations. By observing the behavior of the model, it was found that there are certain phase-space trajectories which the system follows immediately prior to flameout. A controller was developed which detects when the model begins to follow these mediating trajectories and then applies a small perturbation to the mass flow rate of the mixture to push the system onto a different trajectory. Using this strategy, In *et al.* were also able to extend the operating regime of the model to include shorter flow times.

The objective of the current study is to apply nonlinear control to an experimental pulsed combustor to dampen the combustion instabilities which develop at lean conditions and extend the practical operating regime. Emission levels of unburned hydrocarbons and NO_x are monitored to help determine the effectiveness of the controller.

Experimental system

The pulsed combustor used in this study (shown schematically in **Figure 1**) is based upon the design of an experimental thermal pulse combustor developed at METC in which the mixture is introduced at a constant flow rate and combustion is self-sustained by heat transfer from the hot combustor walls (Richards *et al.* 1991). The design consists of a main body, comprised of a 52-cc mixing chamber and a 295-cc combustion chamber, and an acoustically coupled 0.9-m-long tailpipe. The mixing chamber and combustion chamber are separated by a square-celled honeycomb ceramic flameholder that provides a stable location onto which the flame can anchor and acts as a heat source which helps sustain the combustion reaction after start-up.

A spark plug installed in the combustion chamber immediately downstream of the ceramic flameholder is used to ignite the mixture during start-up. The energy released by the combustion reaction heats the combustor walls and the ceramic flameholder which in turn heat the incoming fresh charge. Eventually, the combustor is heated to the point that enough energy is being supplied by the flameholder and combustor walls to auto-ignite the fresh charge and initiate a self-sustaining combustion reaction. At this point, the energy supplied by the spark plug is no longer required to sustain combustion; however, for safety concerns and ease of restart after flameout, the spark plug is typically left on during operation.

The piping system for the combustor includes separate supply lines for the fuel (propane) and oxidizer (air). The mass flow rates of primary fuel and air are intended to be maintained constant regardless of the pressure oscillations occurring within the combustion chamber. To accomplish this, the primary fuel and air supply lines each contain a critical-flow orifice press fit into one of the tube fittings. A third supply line installed coaxial to the primary fuel line is used to provide supplemental injections of fuel which serve as the control perturbation. A low-flow piezoelectric valve (Maxtex, model MV-112) capable of cycling at 250 Hz is used to control the supplemental fuel injections. The throughput rate of the valve is determined by the voltage applied and the fuel supply pressure.

The operating condition of the combustor is defined by two operating parameters: the equivalence ratio, ϕ, and a characteristic flow time, τ, which is defined as the ratio of the internal volume of the combustion chamber to the volumetric flow rate of the mixture referenced to standard atmospheric conditions. Because the equivalence ratio and flow time both depend solely upon the mass flow rates of primary fuel and air, the operating condition of the pulsed combustor may be controlled by varying these mass flow rates. Small changes in the mass flow rates of primary fuel and air are accomplished by varying the supply pressures using a pressure regulator installed in each line, upstream of the critical-flow orifice. Larger changes in mass flow rate are achieved by replacing the critical-flow orifices with larger- or smaller-diameter orifices. The amount of supplemental fuel which must be injected to achieve control can be substantial and can significantly change the equivalence ratio of the mixture. The time-averaged mass flow rate of supplemental fuel is used to determine the effective equivalence ratio, $\bar{\phi}$, which would exist were an equivalent amount of fuel introduced at a steady rate via the primary fuel supply.

Analysis of the pressure inside the combustion chamber was found to be an effective method of characterizing the behavior of the system (Edwards *et al.* 1998; Edwards *et al.* 2000; Edwards 2000). A piezoelectric pressure transducer (Kistler, model 206) connected to a static-pressure tap located 4.75 cm downstream of the ceramic flameholder is used to monitor the combustor pressure. Since the transducer is susceptible to thermal damage when exposed to the high temperatures of the combustor wall and exhaust gases, the tap is water-cooled to prevent overheating. A viewing port is installed in the exhaust system which allows the observer

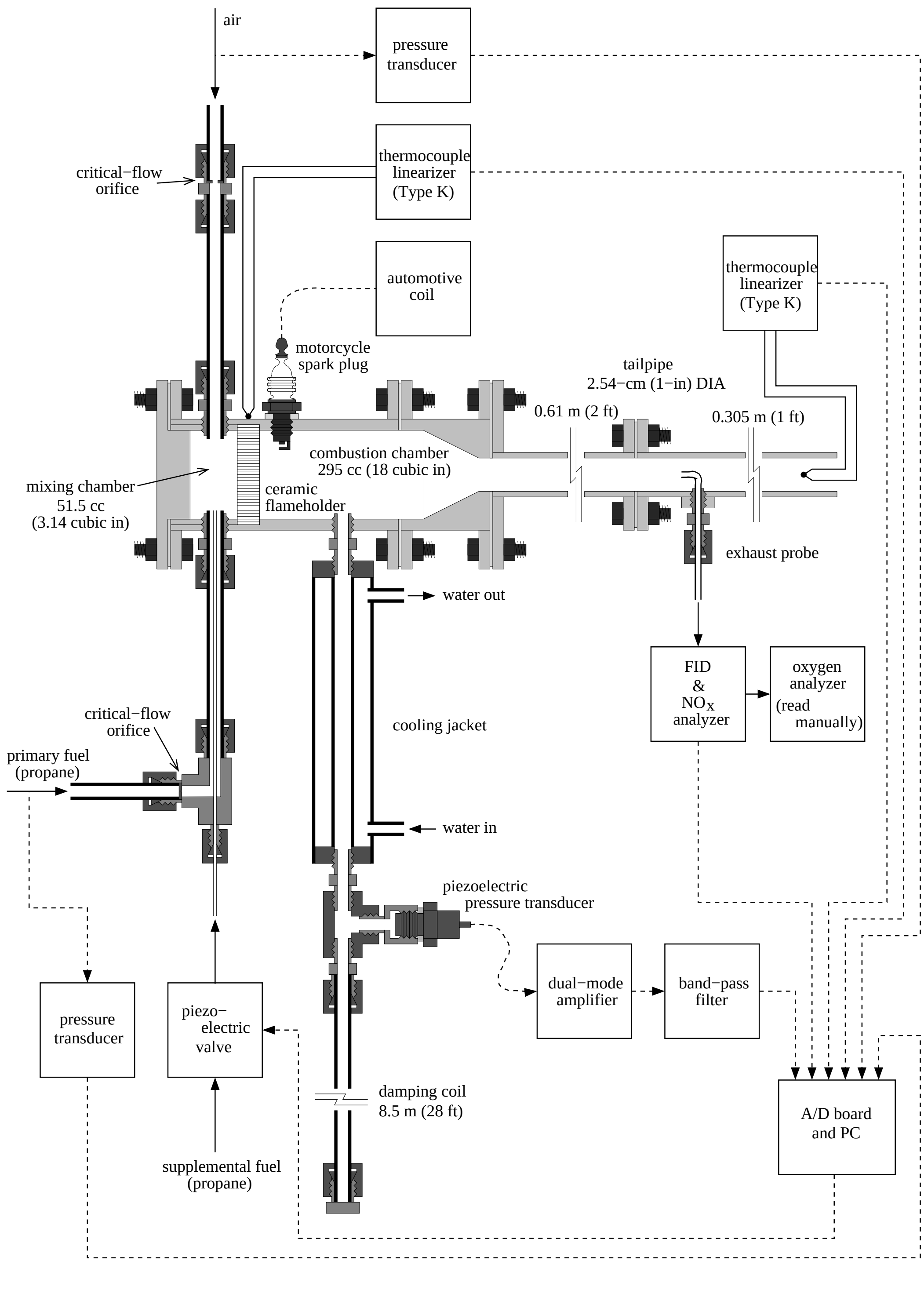


Figure 1: *Schematic of the pulsed-combustor design and instrumentation.*

to look up the tailpipe into the combustion chamber. Correlations can then be made between events seen in the pressure traces and the actual combustion events that are observed to occur inside the combustor.

Shielded, Type-K thermocouples are installed at the tailpipe exit and on the exterior wall of the combustor near the location of the ceramic flameholder to provide approximations of the average exhaust-gas temperature and the combustion-chamber wall temperature at the flame front. While the exhaust-temperature measurement is too slow for cycle-resolved measurements and the wall-temperature reading is below the actual inside wall temperature, the data are adequate to determine when steady state has been achieved and to provide a reference point to help insure accurate duplication of test conditions.

A flame ionization detector, FID, (Thermo Environmental Instruments, model 51) is used to sample the exhaust gases to measure the unburned-hydrocarbon (UHC) emission levels and NO_x emission levels are monitored using an NO_x analyzer (Beckman, model 951). The response time of both instruments is much too slow to make cycle-resolved emission-level measurements; instead, readings are time-averaged over a period of 1–2 min.

Investigating the nature of the instabilities

The behavior of the pulsed combustor was found to be highly dependent upon the equivalence ratio of the fresh charge introduced into the combustion chamber. **Figure 2** illustrates the onset of the combustion instabilities using typical time-series segments of the combustor pressure, measured relative to the time-averaged mean pressure, recorded over a range of equivalence ratios at a flow time of $\tau = 50$ ms. At all operating conditions, the behavior of the system is dominated by large pressure oscillations which occur at the acoustic resonance frequency of the combustor (≈ 100 Hz). At most operating conditions, there is significant variability in pressure behavior from one cycle to the next indicating that there are cycle-to-cycle variations in the quality of the combustion events.

At near-stoichiometric and slightly lean equivalence ratios (**Figures 2(a)–2(e)**), the cyclic variability is relatively small and appears to be random in nature. The degree of cyclic variability increases as the equivalence ratio is brought toward stoichiometry.

As the equivalence ratio approaches the lean flammability limit (**Figures 2(f)–2(i)**), combustion instabilities develop which greatly increase the cyclic variability in combustion quality. Even at moderately lean conditions, the combustion quality occasionally becomes extremely poor resulting in a near misfire (*e.g.,* at $t = 0.65$ s in **Figure 2(f)**). At lower equivalence ratios (**Figure 2(g)**), the pulsed combustor begins to experience intermittent shifts between a relatively stable mode of operation and a mode in which the combustion quality alternates between good and extremely poor in a periodic pattern with a frequency on the order of 10 Hz.

Near the lean flammability limit (**Figures 2(h)** and **2(i)**), the system begins to visit the more stable mode less frequently — if ever. During the poor-quality combustion events, the system actually begins to misfire as confirmed by visual observations in which the flame is seen to be extinguished. Misfire is followed by a rapid recovery during which the magnitude of the combustor-pressure oscillations becomes quite large before the system slowly decays toward another misfire. The frequency of the misfire-and-recovery pattern is on the order of 10 Hz; however, the frequency decreases slightly as the equivalence ratio is decreased. With less energy being released due to the frequent misfires, the combustor wall and the ceramic flameholder begin to cool. Eventually, at the edge of the lean flammability limit, the cooling is sufficient to extinguish the self-sustaining reaction, at which point the pulsed combustor experiences an unrecoverable flameout (**Figure 2(i)** at $t = 0.8$ s), even with the spark plug firing.

The behavioral trends of the pulsed combustor as equivalence ratio is lowered are very similar at other flow times; however, the onset of the combustion instabilities is delayed to lower equivalence ratios as the flow time is lengthened. This seemingly allows the operating range of the pulsed combustor to be extend further toward, and even past, the lean flammability limit simply by decreasing the mass flow rate of the mixture. For example, at $\tau = 50$ ms, flameout occurs at an equivalence ratio of $\phi \approx 0.54$ whereas it is delayed until an equivalence ratio of $\phi \approx 0.30$ at a flow time of $\tau = 100$ ms. The extended operating range of the pulsed combustor at longer flow times is felt to result from poor mixing of the inlet streams. At low mass flow rates, the inlet streams are believed to have insufficient momentum to mix thoroughly resulting in a stratified charge with a localized equivalence ratio which far exceeds the equivalence ratio of the injected mixture. Visual observations through the viewing port in the exhaust system confirm the presence of a “hotspot” on the ceramic flameholder indicating localized combustion.

Figure 3 shows return maps which were created by plotting combustor-pressure measurements originally sampled at 5000 Hz against the third previous pressure measurement. At near-stoichiometric and moderately

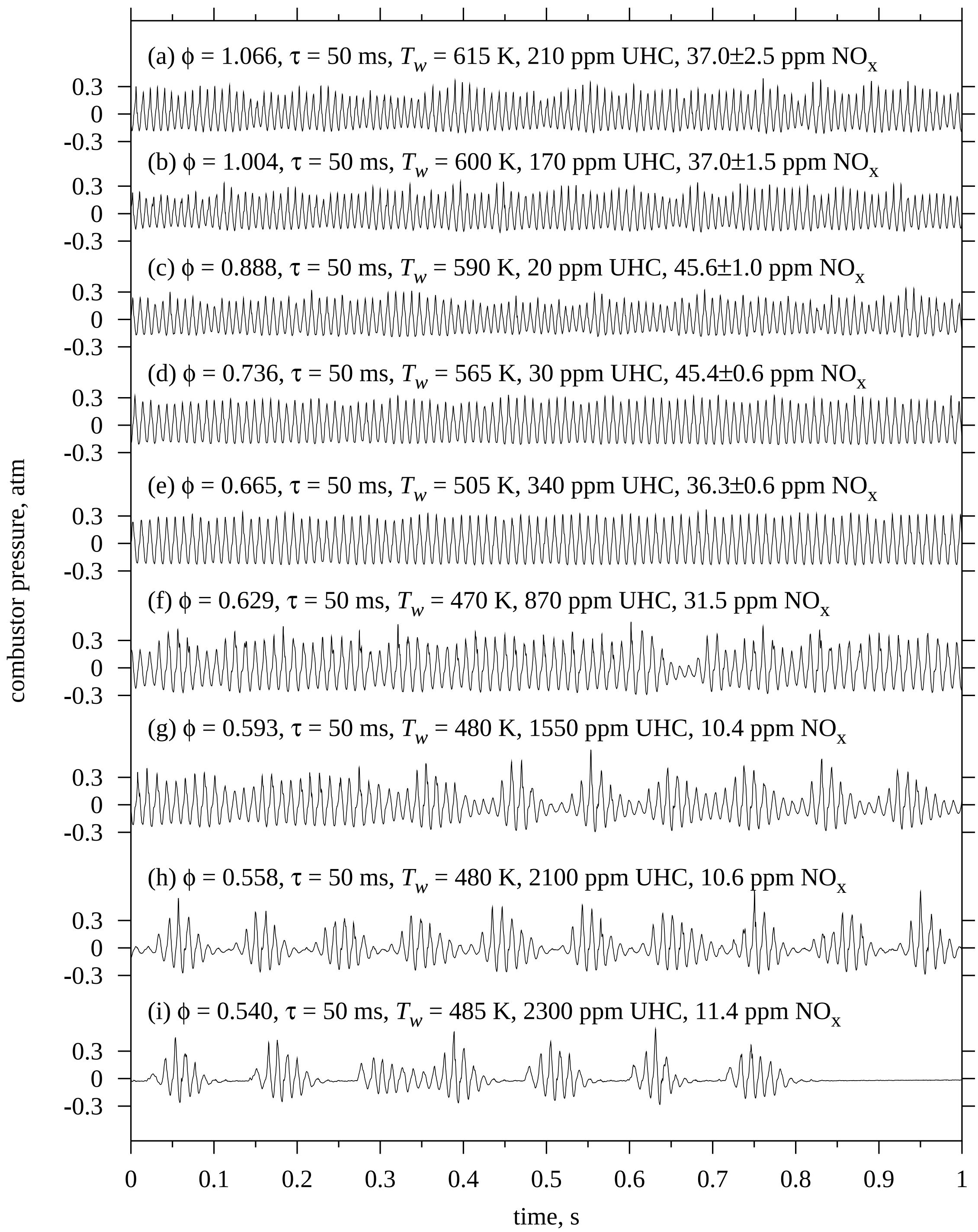
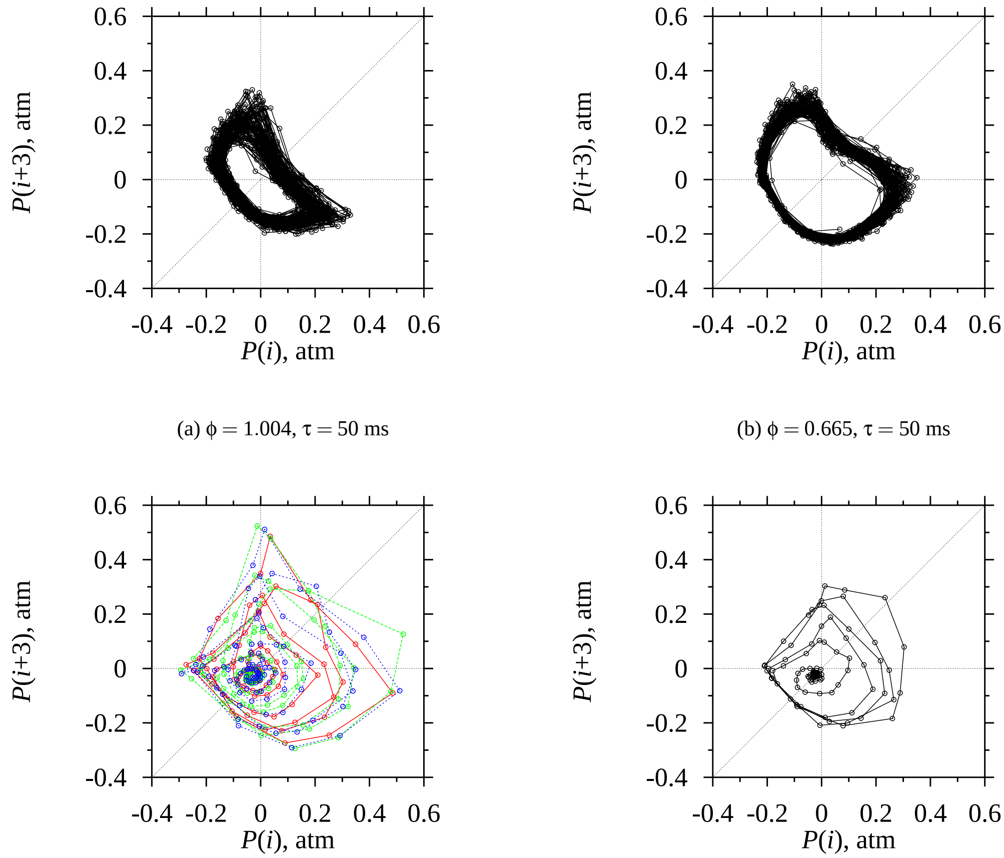


***Figure 2**: Combustor-pressure time-series segments recorded at a flow time of $\tau = 50$ ms showing the onset of combustion instabilities as equivalence ratio is decreased. The signals depict oscillations about the time-averaged mean combustor pressure.*



(a) $\phi = 1.004$, $\tau = 50$ ms

(b) $\phi = 0.665$, $\tau = 50$ ms

(c) $\phi = 0.558$, $\tau = 50$ ms

(d) $\phi = 0.540$, $\tau = 50$ ms

***Figure 3**: Return maps constructed from the combustor-pressure time series over a range of equivalence ratios at a flow time of* $\tau = 50$ *ms.*

lean conditions (**Figures 3(a)** and **3(b)**), the return maps show a noisy stable limit cycle. The combustor-pressure oscillations are produced by acoustic coupling which is an inherently linear process. However, nonlinearities are introduced from numerous sources such as the combustion reaction and mixing of the air, fuel and residual gases. Therefore, it should not be surprising that the combustor-pressure oscillations are quite complex in nature.

In addition to the nonlinearity of the acoustically driven combustor-pressure oscillations, the dynamic behavior of the pulsed combustor is observed to undergo a global bifurcation as the equivalence ratio is reduced toward the lean flammability limit. The return maps shown in **Figures 3(c)** and **3(d)** clearly describe a different behavior than that seen at less-lean conditions. The combustor-pressure follows a trajectory that slowly spirals inward toward misfire before quickly spiraling back outward or collapsing to flameout.

It is conjectured that the combustion instabilities result from nonlinearities inherent to the combustion reaction near the lean flammability limit. Strong combustion events are believed to consume the available fuel inventory. During the subsequent cycles, combustion quality is poor until a sufficient fuel inventory has been restocked. When the restocking of the fuel inventory becomes too slow, insufficient energy is released during the poor-quality combustion events to maintain the combustor walls and the ceramic flameholder at a temperature which is sufficient to maintain the self-sustained combustion reaction. At this point, the pulsed combustor cannot recover from misfire and the system spirals inward toward flameout.

When the pulsed combustor is operating near the global bifurcation point, the system intermittently alternates between the two modes. Due to the nonlinear nature of the system, when a small disturbance pushes the pulsed combustor across the bifurcation point toward one of these modes of operation, it tends to become entrained in that mode until another small disturbance causes the system to deviate back toward the other mode. A control strategy similar to the one developed by In *et al.* (1997) which monitors the the state of the combustor to detect when the behavior begins to transition toward the unstable mode and then applies an appropriate perturbation to drive the system back toward the stable mode should prove effective in dampening the combustion instabilities and extending the practical operating range of the pulsed combustor.

Appying feedback trajectory-correction control

The control scheme shown schematically in **Figure 4** has been developed to take advantage of the nonlinear nature of the system by applying small appropriately timed supplemental fuel injections to hasten the restocking process and push the trajectory of the system back toward the stable mode of operation. The controller monitors the combustor pressure and tallies the average cycle maximum from peak pressure values. A control action is initiated when the peak pressure for a cycle falls below the trigger level which is specified by the operator as some percentage of the average cycle maximum. After an operator-specified triggering delay, the control perturbation is initiated. The amount of fuel injected with each control perturbation depends upon the pulse duration and the settings of the piezoelectric valve used to supply the fuel.

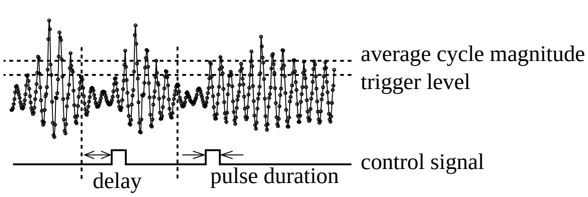


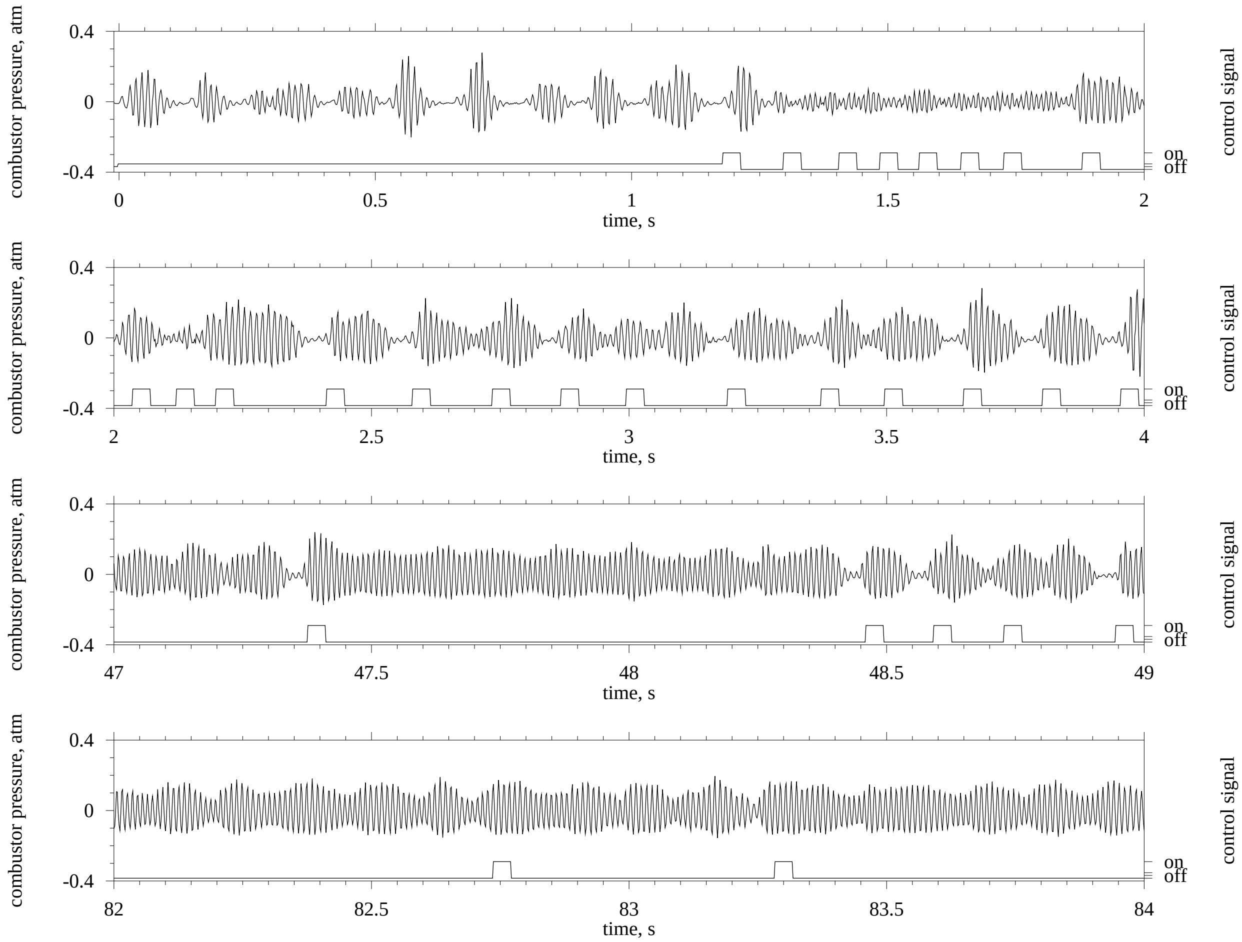
***Figure 4**: Graphical depiction of the control algorithm.*

Figure 5(a) shows four time-series segments of the combustor pressure and control signal collected over various intervals as the controller is activated with the pulsed combustor operating at an equivalence ratio of $\phi = 0.427$ and a flow time of $\tau = 75$ ms. At this operating condition, the pulsed combustor experiences frequent and severe misfire and will eventually flame out unless control is applied. The controller is turned on at time $t = 0$ with a trigger level of 50% of the average cycle maximum, a triggering delay of 42 ms and a pulse duration of 33 ms. The controller enters a brief learning period to determine the average cycle maximum before any control actions can be applied. The uncontrolled behavior is apparent in the first time-series segment in **Figure 5(a)** during the learning period (from $t = 0$ to $t = 1.2$ s).

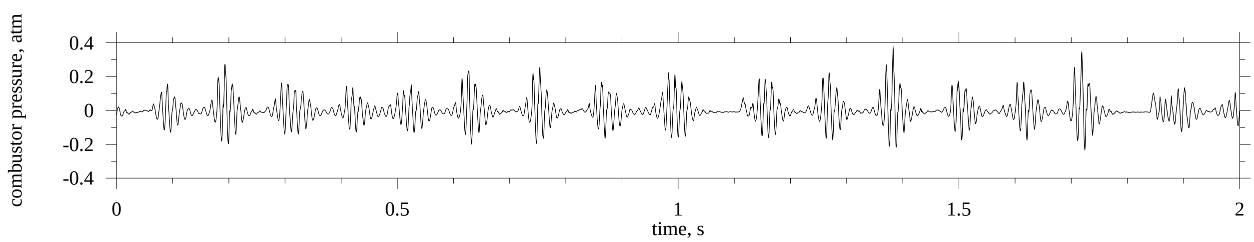
The first control action is initiated at $t \approx 1.2$ s, momentarily driving the system into an erratic, low-amplitude behavior from which the combustor quickly recovers. Initially, control actions are frequent and significant amounts of supplemental fuel are added resulting in an effective equivalence ratio of $\bar{\phi} \approx 0.61$. Within 4 s (second segment in **Figure 5(a)**), the combustor performance is already improving, as evidenced by the slightly longer periods of stable operation without misfire. As the combustor becomes entrained in a more stable operating mode, the wall temperature increases, thereby further stabilizing the behavior. Control actions are required less frequently and within 90 s are only occasionally necessary to keep the system entrained in the stable operating mode indefinitely (fourth segment in **Figure 5(a)**). At this point, the injections of supplemental fuel result in an effective equivalence ratio of $\bar{\phi} \approx 0.45$. There are still low-frequency fluctuations in the magnitude of the combustor pressure due to the combustion instabilities, but the magnitude of the fluctuations have been greatly reduced and misfire has been all but eliminated.

It is important to note that the behavior of the pulsed combustor is much more efficient with feedback control than if a steady flow of primary fuel were used to yield a similar equivalence ratio. **Figure 5(b)** shows a combustor-pressure time-series segment collected at an equivalence ratio of $\phi = 0.450$. The enhanced combustion quality with control is readily evident. At $\phi = 0.450$, the behavior of the combustor is not noticeably different than the uncontrolled behavior at an equivalence ratio of $\phi = 0.427$, and flameout still occurs. With control, the UHC emission levels are greatly reduced from 1600 ppm at $\phi = 0.450$ to approximately 80 ppm with $\bar{\phi} \approx 0.45$.

Table 1 demonstrates the effectiveness of the controller in reducing UHC emission levels at lean con-



(a) $\bar{\phi} \approx 0.45$ ($\phi = 0.427$), $\tau = 75$ ms, $T_w = 520$ K, ~ 80 ppm UHC



(b) $\phi = 0.45$, $\tau = 75$ ms, $T_w = 450$ K, ~ 1600 ppm UHC

***Figure 5**: Time-series segments of combustor pressure showing the application of control and the effect of injecting an equivalent amount of fuel via the primary-fuel supply. Control variables used in this example: trigger level = 50%; delay = 42 ms; pulse duration = 33 ms.*

***Table 1**: UHC emission levels with and without control.*

(a) $\tau = 50$ ms			
ϕ	UHC emission levels without control (ppm)	$\bar{\phi}$	UHC emission levels with control (ppm)
0.665	340±50	0.687	100±20
0.647	540±100	0.669	100±20
0.629	870±100	0.663	125±10
0.611	1150±100	0.656	130±10
0.593	1550±200	0.672	130±20
0.575	1750±200	0.688	160±30
0.558	2100±300	0.670	190±40
0.540	2300±300	0.653	220±50
0.522	—	0.635	400±150

(b) $\tau = 75$ ms			
ϕ	UHC emission levels without control (ppm)	$\bar{\phi}$	UHC emission levels with control (ppm)
0.487	105±20	0.487	60±20
0.469	980±200	0.469	60±10
0.452	1600±300	0.452	160±30
0.435	1600±300	0.450	125±25
0.418	—	0.435	250±200
0.400	—	0.430	500±200
0.382	—	0.425	700±100
0.366	—	0.410	850±100

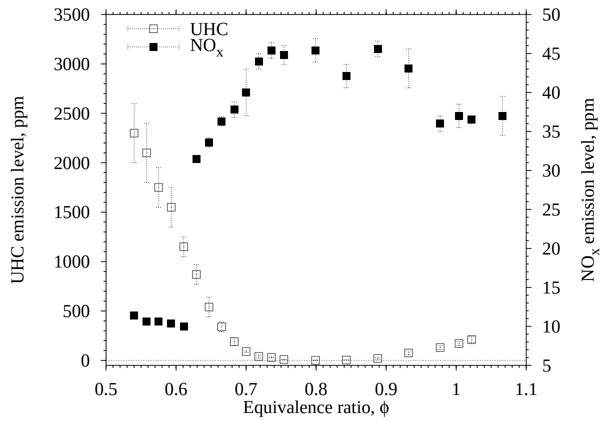


***Figure 6**: UHC and NO_x emission levels detected in the exhaust gases of the pulsed combustor over a range of equivalence ratios at a flow time of $\tau = 50$ ms without control.*

***Table 2**: NO_x emission levels with and without control.*

$\tau = 50$ ms			
ϕ	NO_x emission levels without control (ppm)	$\bar{\phi}$	NO_x emission levels with control (ppm)
0.593	10.6	0.672	10.3±0.7
0.575	10.6	0.688	10.0±0.8
0.558	10.6	0.670	10.1±0.7
0.540	11.4	0.653	10.1±0.7

ditions. For a flow time of $\tau = 50$ ms, it is clear that the controller significantly reduces the UHC emission levels. For example, at an equivalence ratio of $\phi = 0.647$, the UHC emission levels are reduced from 540 ± 100 ppm without control to 100 ± 20 ppm with control while the effective equivalence ratio is only raised to $\bar{\phi} = 0.669$. Comparing this value to the UHC emission levels for the uncontrolled case for $\phi = 0.665$, it is clear that the controller provides a greater reduction in UHC emission levels than would be achieved were the same amount of fuel introduced at a steady rate through the primary fuel supply.

As the equivalence ratio is reduced further, even greater reductions in UHC levels are seen. For example, at an equivalence ratio of $\phi = 0.540$, UHC emission levels are reduced by a factor of ten with control. At this condition, control actions are required more frequently which greatly increases the effective equivalence ratio (to $\bar{\phi} = 0.653$ in this example); however, the reduction in UHC emission levels is still about twice that which would be achieved by injecting a similar amount of fuel at a steady rate through the primary fuel supply (based upon interpolation from data at $\phi = 0.665$ and $\phi = 0.647$).

The reductions are even more dramatic at a flow time of $\tau = 75$ ms. At an equivalence ratio of $\phi = 0.452$, the UHC emission levels are reduced by a factor of ten using control perturbations which are required so infrequently that there is no measurable increase in effective equivalence ratio. With control, the operating regime of the pulsed combustor can be extended further lean to equivalence ratios at which the system would otherwise flame out. For example, with control, the combustor will operate at an equivalence ratio of $\phi = 0.366$. While the UHC emission levels are quite high at this condition (850 ± 100 ppm), note that the combustor would flame out were an equivalent amount of fuel injected through the primary fuel supply.

As shown in **Figure 6**, without control, NO_x emission levels substantially decrease as equivalence ratio is lowered. There is a discontinuous drop in NO_x once the combustion instabilities begin to develop due in part to the frequent lower combustor temperatures and misfire. By increasing combustion efficiency and the combustor temperature, it was feared that the controller would negate the reductions in NO_x emission levels. However, as shown in **Table 2**, NO_x emission levels remain approximately the same with the current control strategy.

Summary

It has been shown that the behavior of the pulsed combustor is driven by two different mechanisms. Large-amplitude pressure oscillations which occur at the acoustic frequency of the system are the result of acoustic coupling with the tailpipe. These oscillations are nonlinear in nature due to interactions with the combustion reaction and turbulent mixing effects. In addition to the acoustically driven oscillations, the nonlinear nature of the combustion reaction leads to a global bifurcation in system dynamics as the equivalence ratio of the fresh mixture approaches the lean flammability limit. At these conditions, the combustion quality of the current cycle has a significant effect upon the quality of several subsequent cycles. High-quality combustion events consume the available fuel inventory which must then be restocked. During the restocking process, the combustion events are of poor quality, and at extremely lean conditions, the combustor will misfire. If the restocking process is too slow, the combustor walls and ceramic flameholder will begin to cool until self-sustained combustion can no longer be maintained and the pulsed combustor flames out.

A feedback control strategy has been developed which monitors the peak pressure of each cycle to detect when the available fuel inventory has been consumed and the pulsed combustor begins to experience poor-quality combustion events while the fuel inventory is restocked. The controller then injects a small amount of supplemental fuel to hasten the restocking process and push the trajectory of the system back toward the stable operating mode. The control strategy has proven to be very effective at dampening the combustion instabilities and thereby reducing the UHC emission levels and extending the operating regime of the combustor further toward the lean flammability limit without incurring an increase in NO_x emission levels.

The trial-and-error method used to choose the control variables in this study is far from efficient. A neural-network algorithm could be developed to improve selection of the control variables and refine control. Using this method, it is suspected that control would be more efficient near the lean flammability limit. Nonetheless, the current study has proven that a nonlinear control strategy is indeed effective in reducing cyclic variability and extending the practical operating regime of a pulsed combustor.

References

Arpaci, V. S., J. E. Dec, and J. O. Keller (1991). Heat transfer in pulse combustor tailpipes. *Proceedings of the International Symposium on Pulsating Combustion, Volume 2*. Sandia National Laboratories and Gas Research Institute. Paper F-3. 5-8 August, Monterey CA, USA.

Daw, C. S., J. F. Thomas, and G. A. Richards (1992). Modeling deterministic chaos in thermal pulse combustion. *1992 Technical Meeting of the Central States Section of the Combustion Institute*. 27-28 April.

Daw, C. S., J. F. Thomas, G. A. Richards, and L. L. Narayanaswami (1994). Experimental evidence for deterministic chaos in thermal pulse combustion. *1994 Technical Meeting of the Central States Section of the Combustion Institute*, pp. 439–444. 5-7 June, Madison WI, USA.

Daw, C. S., J. F. Thomas, G. A. Richards, and L. L. Narayanaswami (1995). Chaos in thermal pulse combustion. *Chaos 5*(4), 662–670.

Dec, J. E. and J. O. Keller (1989). Pulse combustor tail-pipe heat-transfer dependence on frequency, amplitude, and mean flow rate. *Combustion and Flame 77*, 359–374.

Edwards, K. D. (2000). *Application of nonlinear feedback control to enhance combustion quality in a pulsed combustor*. Ph. D. dissertation, University of Tennessee, Knoxville.

Edwards, K. D., C. E. A. Finney, K. Nguyen, and C. S. Daw (1998). Use of symbol statistics to characterize combustion in a pulse combustor operating near the fuel-lean limit. *1998 Technical Meeting of the Central States Section of the Combustion Institute*, pp. 385–390. 31 May - 2 June, Lexington KY, USA.

Edwards, K. D., C. E. A. Finney, K. Nguyen, and C. S. Daw (2000). Application of nonlinear feedback control to enhance the performance of a pulsed combustor. *2000 Technical Meeting of the*

Central States Section of the Combustion Institute, pp. 249–254. 16-18 April, Indianapolis IN, USA.

Gemmen, R. S., J. O. Keller, and V. S. Arpaci (1991). Heat/mass transfer from a cylinder in the strongly oscillating flow of a pulse combustor tailpipe. *Proceedings of the International Symposium on Pulsating Combustion, Volume 2*. Sandia National Laboratories and Gas Research Institute. Paper F-5. 5-8 August, Monterey CA, USA.

Hanby, V. I. (1969). Convective heat transfer in a gas-fired pulsating combustor. *Journal of Engineering for Power 91*, 48–52.

In, V., M. L. Spano, J. D. Neff, W. L. Ditto, C. S. Daw, K. D. Edwards, and K. Nguyen (1997). Maintenance of chaos in a computational model of a thermal pulse combustor. *Chaos 7*(4), 605–613.

Keller, J. O., T. T. Bramlette, P. K. Barr, and J. R. Alvarez (1994). NO_x and CO emissions from a pulse combustor operating in a lean premixed mode. *Combustion and Flame 99*, 460–466.

Keller, J. O. and I. Hongo (1990). Pulse combustion: The mechanisms of NO_x production. *Combustion and Flame 80*, 219–237.

Margolis, S. B. (1994). The nonlinear dynamics of intrinsic acoustic-oscillations in a model pulse combustor. *Combustion and Flame 99*, 311–322.

Rhode, M. A., R. W. Rollins, A. J. Markworth, K. D. Edwards, K. Nguyen, C. S. Daw, and J. F. Thomas (1995). Controlling chaos in a model of thermal pulse combustion. *Journal of Applied Physics 78*(4), 2224–2232.

Richards, G. A., G. J. Morris, D. W. Shaw, S. A. Keeley, and M. J. Welter (1991). Thermal pulse combustion. *Proceedings of the International Symposium on Pulsating Combustion, Volume 1*. Sandia National Laboratories and Gas Research Institute. Paper A-11. 5-8 August, Monterey CA, USA.

Sterling, J. D. (1993). Nonlinear analysis and modelling of combustion instabilities in a laboratory combustor. *Combustion Science and Technology 89*, 167–179.

Sterling, J. D. and E. E. Zukoski (1991). Nonlinear dynamics of laboratory combustor pressure oscillations. *Combustion Science and Technology 77*(4-6), 225–238.